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ESTIMATING A SEISMIC STATION'S DETECTION CAPABILITY
FROM NOISE. APPLICATION TO VLPE STATIONS

Rudolf Unger

Texas Instruments, Incorporated

Prepared for:

Advanced Research Projects Agency
Air Force Technical Applications Center

22 October 1974

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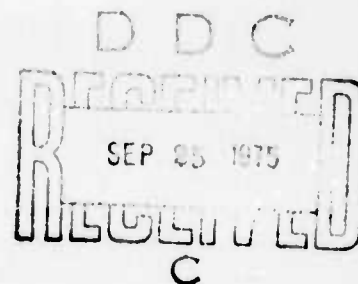
VELA NETWORK EVALUATION AND AUTOMATIC PROCESSING RESEARCH

Prepared by
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Alexandria, Virginia 22314

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capabilities, and the parameters involved, are specified. The analysis shows that results obtained from this method should differ little from maximum likelihood direct method estimates. Application of this method to stations of the Very Long Period Experiment (VLPE) confirms the analytical results. As few as 30 reliable daily noise samples yield good 50% detection threshold estimates.

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ABSTRACT

Since the detection of a seismic event depends on the signal-to-noise ratio at a receiving site or station one may derive detection threshold magnitudes from the ambient noise levels. The statistical distributions of either peak noise amplitudes or RMS noise levels then determine the probability of detecting an event of a given magnitude. The conditions for the validity of this method of estimating detection capabilities, and the parameters involved, are specified. The analysis shows that results obtained from this method should differ little from maximum likelihood direct method estimates. Application of this method to stations of the Very Long Period Experiment (VLPE) confirms the analytical results. As few as 30 reliable daily noise samples yield good 50% detection threshold estimates.

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SECTION I

INTRODUCTION

In monitoring earthquakes and underground nuclear explosions one is interested in the detection capability of a seismic station, array or network. This detection capability is usually expressed as the probability of detecting a seismic event of a given magnitude in a certain region. One method to determine the detection capability is to compute the percentage of events actually detected for each of several magnitudes. The maximum likelihood approach to this so-called direct method fits a Gaussian probability of detection curve to the detection percentages (Ringdal, 1974). The direct method requires a large population of events and consequently, establishing the detection capability of a station, array or network takes considerable time.

Another method estimates the detection capability based on the ambient noise levels at a station (Lacoss, 1969; Harley, 1971; Harley and Heiting, 1972). In this method it is assumed that an event signal can be detected when its maximum amplitude exceeds that of the surrounding noise by a certain margin; e.g., 3 dB. We may then assign a magnitude to the maximum noise amplitude (A) occurring in a certain time gate, using some period (T) and some epicentral distance (Δ). For surface waves this magnitude is (Harley, 1972):

$$M_{s_{noi}} = \log A/T + \log \Delta + 1.12 + C_1 \quad (I-1)$$

where $\log \Delta + 1.12$ represents the distance correction, and C_1 is a detection criterion margin, e.g., $C_1 = 0.15$ for a 3 dB margin. The

probability of detecting an event of given magnitude $M_{s\text{sig}} = x$ then is assumed to be given by the probability that the noise magnitude is less than or equal to x :

$$P(\text{det } M_{s\text{sig}} = x) = P(M_{s\text{noi}} \leq x) . \quad (\text{I-2})$$

Usually, collecting noise data (for instance, an ensemble of daily noise samples) is a much faster process than gathering data from actual events, and if noise amplitude distributions are stationary with time the detection capability can be established in a relatively short time period. This is particularly important in evaluating new or proposed seismic station sites.

In the above formulae some of the parameters involved have not been studied in detail, and so their effects in estimating detection capabilities have not been fully specified. Also, Lambert observed from his Very Long Period Experiment (VLPE) data that the noise peak-amplitude-over-RMS ratio at the seismometer output was relatively constant for the individual stations as well as from station to station (Lambert et al., 1973), and he raised the possibility that 50% detection thresholds may be estimated from average RMS noise levels.

The analytical and empirical investigation of the above topics are the subject of this report. Analytically, the probability of detection is developed into validity conditions for the application of detectability estimation methods, and differences between the estimation from noise and direct estimation are discussed. Empirically, the method of estimating detection capabilities from noise assuming a Gaussian probability of detection is applied to stations of the VLPE network, and the results are compared with maximum likelihood direct estimates for those stations. The study has been confined to surface wave detection threshold estimates derived from the vertical Rayleigh wave component, but could be extended to include horizontal Rayleigh wave, Love wave, and/or bodywave detection threshold estimation.

Section II of this report contains the analytical examination of the subject. The empirical evaluation, including a description of the available data, a discussion of the noise data editing process, the detection threshold estimation procedure, and the discussion of the results, are given in Section III. The study is summarized in Section IV. Reference material is found in Section V, and further details used in this study are given in the appendices.

SECTION II

ANALYTICAL DEVELOPMENT

In this section the topics mentioned in the introduction are examined analytically. First, some surface-wave magnitude relationships are established. Next, the probability of detecting an event of given reference magnitude is developed into the probability that the event magnitude exceeds a certain noise magnitude. The elements of this noise magnitude are specified and examined, and their effects on detection capability estimation are discussed. Via a brief discussion of detection capability expressions, this leads to conditions and rules for the validity of the application of detection capability estimation methods, and to specification of their differences. It is then shown that RMS noise levels may be used in combination with the maximum amplitude-over-RMS ratio to replace maximum noise amplitudes in estimating detectabilities from noise. Also, some considerations in averaging logarithmic terms are given. The last part of this section summarizes the analytical results.

A. SURFACE-WAVE MAGNITUDE RELATIONSHIPS

For a clear understanding of the various surface-wave magnitude values involved in detection capability estimation these magnitudes are defined below, and their mutual relationships are given.

The surface-wave magnitude for a station or array is given by the formula (Harley, 1972):

$$M_s = \log \frac{A}{T} + \log \Delta + 1.12 \quad (\text{II-1})$$

where:

- A is the maximum peak-to-peak event signal amplitude in $m\mu$ ground motion at the station, i. e., the instrument corrected, or seismometer input amplitude;
- T is the period of the maximum amplitude A in sec;
- Δ is the epicentral distance in degrees.

In many cases "the" surface wave magnitude is understood to be the magnitude derived from the maximum amplitude with its period near 20 seconds. Actual detections, however, may be established from amplitudes with a different period. The empirically determined magnitude differences due to period effects are rather constant, so we will use the relationship:

$$M_s(T) = M_s(20) - d(T) \quad (11-2)$$

where:

- $M_s(T)$ is the T-sec magnitude
- $M_s(20)$ is the 20-sec station magnitude
- $d(T)$ is the station magnitude difference due to period T.

Marshall and Basham (1972) found a 30-sec magnitude difference of 0.30 for Eurasian continental paths. Lambert and Becker (1973) computed a least-mean square $d(30)$ value for Northern hemisphere, Eurasian continental path VLPE stations, from both winter, and summer events; the winter value was 0.33, the summer value 0.29, both in good agreement with Marshall and Basham.

Due to radiation pattern and propagation path effects each station or array will measure a different magnitude for one given event. A T-sec

network surface-wave magnitude is obtained by averaging, for a given T , the T -sec magnitudes measured at the stations and arrays of the network concerned:

$$\bar{M}_{s_{\text{net}}}(T) = \frac{1}{N} \sum_{i=1}^N M_{s_i}(T) \quad (\text{II-3})$$

where i is the station index and N is the number of network stations. A network may consist of, for instance, all VLPE stations (the VLPE network), or may include the LASA, NORSAR, or ALPA arrays.

To evaluate the performance of a station or network one must have an independent, preferably global, reference magnitude available. This reference magnitude could be established in the same manner as the network magnitude from world wide stations and arrays. However, no current global network reports sufficient M_s data to justify averaging. A reference surface wave magnitude, therefore, may be obtained by converting a global bodywave magnitude to a 20-sec surface-wave magnitude. For the VLPE network, for example, the following conversion formula was established by least-mean square fitting of reference bodywave magnitudes and VLPE station 20-sec surface-wave magnitudes (Lambert and Becker, 1973):

$$\bar{M}_{s_{\text{ref}}}(20) = 1.20 \bar{m}_b - 1.74 \quad (\text{II-4})$$

where \bar{m}_b represents the reference bodywave magnitude.

The station magnitude deviation from the 20-sec network or reference magnitude is denoted by b in the expression

$$M_s(20) = \bar{M}_s(20) + b \quad (\text{II-5})$$

in which $M_s(20)$ and b are random variables. The mean deviation \bar{b} is called the station magnitude bias.

Combining the relationships (II-2) and (II-5) we obtain:

$$M_s(T) = \bar{M}_s(20) - d(T) + b \quad (II-6)$$

where:

$M_s(T)$ is the T-sec station magnitude;

$\bar{M}_s(20)$ is the 20-sec reference magnitude;

$d(T)$ is the magnitude difference due to period T;

b is the station magnitude deviation.

B. THE PROBABILITY OF DETECTION AS A FUNCTION OF EVENT MAGNITUDE

One of the criteria used by the analyst in the mechanism of detecting seismic events is that an event is detected when at the seismometer output the maximum signal amplitude exceeds the maximum amplitude of the surrounding noise by a certain margin (e.g., 3 dB). Other criteria, such as dispersion and arrival time may also play a role in the analyst's decision (e.g., Lambert et al., 1973). To formulate the detection capability of a seismic station, we focus on the first mentioned criterion, which we will call the signal-to-noise ratio (SNR) criterion. The probability of detecting events then is determined by the probability distribution of maximum noise amplitudes. The probability of detecting an event given its 20-sec reference surface-wave magnitude may be expressed as the probability that given the event magnitude, its signal amplitude exceeds the maximum noise amplitude:

$$P(\det \bar{M}_s(20) = x) = P(A_{S_{out}} \geq C_2 \cdot A_{N_{out}}) \quad (II-7)$$

where

$\bar{M}_s(20)$ is the 20-sec reference magnitude for the event;

x is the given reference magnitude value;

- $A_{S_{out}}$ is the maximum peak-to-peak event signal amplitude at the seismometer output;
- $A_{N_{out}}$ is the maximum peak-to-peak noise amplitude at the seismometer output within some timegate;
- C_2 is the detection margin factor, e.g., $C_2 = 1.4$ for a 3 dB margin.

However, we would like to express the probability of detection as a probability of magnitude inequalities. Since the logarithmic function is monotonically increasing we may replace the amplitudes by their base ten logarithms and introduce the parameters signal period (T) and epicentral distance (Δ) to establish a relationship reflecting the event's station magnitude. Since the magnitude must represent the true ground motion at the station, we first correct for the instrument response:

$$A_{S_{in}} = \frac{A_{S_{out}}}{G(T)} \quad (II-8)$$

where:

$A_{S_{in}}$ is the peak-to-peak event signal amplitude at the seismometer input in $m\mu$ true ground motion corresponding to the maximum peak-to-peak output amplitude;

T is the period of the maximum output amplitude;

$G(T)$ is the instrument response for the period T .

With the modifications mentioned above we then obtain:

$$\begin{aligned} F(\det \bar{M}_s(20) = x) &= P(\log \frac{A_{S_{in}}}{T} + \log \Delta + 1.12 \\ &\geq \log \frac{A_{N_{out}}}{T \cdot G(T)} + \log \Delta + 1.12 + C_1) \end{aligned} \quad (II-9)$$

where $C_1 = \log C_2$.

The left-hand side in the inequality now represents the T-sec station surface-wave magnitude, $M_s(T)$. Similarly, we may denote the right-hand side by a magnitude, and we will call this the station "noise magnitude", M_N :

$$M_N = \log \frac{A_{N_{out}}}{T \cdot G(T)} + \log \Delta + C \quad (II-10)$$

where $C = 1.12 + C_1$. The probability of detection then becomes

$$P(\det \bar{M}_s(20) = x) = P(M_s(T) \geq M_N). \quad (II-11)$$

Note that the noise magnitude is not a magnitude in the usual sense because the parameters T , $G(T)$ and Δ refer to the event rather than to the noise, and because the noise magnitude contains a detection margin term.

Using the relationship between the T-sec station magnitude and the 20-sec reference magnitude (II-6) we obtain:

$$P(\det \bar{M}_s(20) = x) = P(\bar{M}_s(20) - d(T) + b \geq M_N). \quad (II-12)$$

We may now use the substitution $\bar{M}_s(20) = x$ and re-arrange the terms:

$$P(\det \bar{M}_s(20) = x) = P(M_N - d(T) - b \leq x). \quad (II-13)$$

Thus, the probability of detecting an event, given its 20-sec reference magnitude may be expressed as the probability that the random variable (r.v.) $M_N + d(T) - b$ is less than or equal to the given reference magnitude. The statistics of all terms constituting this random variable, therefore, will determine the statistical model on which detection capability estimation methods may be based.

For convenience in the further discussion we will use the symbol M for the r.v. $M_N + d(T) - b$. Furthermore, we denote the

probability distribution function of the r.v. M by F_M and obtain the final relationships:

$$P(\det \bar{M}_s(20) = x) = P(M \leq x) = F_M(x) \quad (II-14)$$

where

$$M = M_N + d(T) - b. \quad (II-15)$$

C. THE RANDOM VARIABLE $M = M_N + d(T) - b$

We will now investigate the statistical characteristics of each term contributing to the r.v. $M = M_N + d(T) - b$. For this we use the expression for M_N given in Equation (II-10) and slightly re-arrange its variables:

$$M = \log A_{N_{out}} - \log T - \log G(T) + \log \Delta + C + d(T) - b. \quad (II-16)$$

The statistical investigation thus will focus on the terms $\log A_{N_{out}}$, $\log T$, $\log G(T)$, $\log \Delta$, $d(T)$, b , and any linear combination of these. The term C is a constant and needs no further examination.

The investigation of the r.v. M and its constituents is given in Appendix A. Histograms of actual data are shown for the parameters $\log A_{N_{out}}$, $\log T$, and $\log \Delta$. The data for $A_{N_{out}}$ reflect noise samples used in the empirical evaluation of this study; the T and Δ data were taken from events establishing the maximum likelihood direct detection capability estimates (Lambert et al., 1973) also used in this evaluation. Furthermore, a relationship between the period dependent terms T , $G(T)$, and $d(T)$ is derived, and the statistics of the station magnitude variation are discussed.

The results show to what extent each of the terms involved, and their linear combination, could be considered a normal r.v.. It is concluded that if the range of relative distances is small ($0.7 < \Delta/\Delta_0 < 1.6$, where Δ_0 is the geometric mean distance), and based only on visual

inspection of the histograms concerned, the r.v. M may be considered to be approximately normally distributed for most stations. Thus, for a narrow relative distance range, the probability distribution of detecting an event of given reference magnitude may be considered to be approximately Gaussian in most cases.

Furthermore, the terms $\log G(T)$ and $d(T)$ were found to be approximately linear with $\log T$. If the parameters T , Δ , and b are uncorrelated, then the variance of the r.v. M is approximately:

$$\sigma_M^2 = \sigma_{\log A}^2 + (\beta - \alpha - 1)^2 \sigma_{\log T}^2 + \sigma_{\log \Delta}^2 + \sigma_b^2 \quad (\text{II-17})$$

where

- σ_M is the standard deviation of the r.v. M ;
- $\sigma_{\log A}$ is the standard deviation of $\log A_{N_{out}}$;
- $\sigma_{\log T}$ is the standard deviation of $\log T$;
- $\sigma_{\log \Delta}$ is the standard deviation of $\log \Delta$;
- σ_b is the standard deviation of the station magnitude bias;
- α is the slope of the instrument response between 20-sec and 30-sec periods ;
- β is the slope of the $d(T)$ versus $\log T$ curve between 20-sec and 30-sec periods.

Empirical values for the above parameters result in a standard deviation range for the r.v. M of

$$0.39 \leq \sigma_M \leq 0.55 \quad (\text{II-18})$$

over the stations concerned. The actual value of σ_M depends on the station's ambient noise, its instrument response, the type of propagation path between the event epicenters and the station (continental or mixed continental-oceanic paths), and the relative distance range. Evidently, for some stations the

value σ_M is higher than the standard deviation ($\sigma = 0.4$) arrived at by Lambert et al. (1973).

D. DETECTION CAPABILITY EXPRESSIONS

To describe the detection capability of a seismic station, array or network one considers the reference magnitude values for which the probability of detection is a certain percentage. These reference magnitude values are called detection thresholds. For instance, the 50% and 90% detection thresholds, denoted by x_{50} and x_{90} , respectively, are given by

$$P(\det \bar{M}_s(20) = x_{50}) = 0.50 \quad (\text{II-19})$$

and

$$P(\det \bar{M}_s(20) = x_{90}) = 0.90. \quad (\text{II-20})$$

Equation (II-14) translates this into:

$$P(M \leq x_{50}) = F_M(x_{50}) = 0.50 \quad (\text{II-21})$$

and

$$P(M \leq x_{90}) = F_M(x_{90}) = 0.90. \quad (\text{II-22})$$

Thus, if the probability distribution function of the r.v. M , $F_M(x)$, is known, either in tabulated, graphical, or closed mathematical form, the desired threshold value is found from the argument of $F_M(x)$. For instance, if M is a Gaussian r.v., then (Papoulis, 1965):

$$F_M(x) = 0.5 + \text{erf} \left(\frac{x - \mu_M}{\sigma_M} \right) \quad (\text{II-23})$$

where:

μ_M is the mean of the r.v. M ;

σ_M is the standard deviation of the r.v. M .

For this case, the 50% and 90% detection thresholds are:

$$x_{50} = \mu_M \quad (II-24)$$

and

$$x_{90} = \mu_M + 1.28 \sigma_M . \quad (II-25)$$

It should be pointed out, however, that M need not be Gaussian to determine the 50% detection threshold as the mean of M . It suffices that the probability density function of M be even about its mean to satisfy Equation (II-24).

E. DETECTION CAPABILITY ESTIMATION

In Subsection II-B we described mathematically the SNR detection mechanism used by the analyst in detecting seismic events, and we studied the statistics involved. We arrived at a general detection formula which is independent of the method of detection capability estimation actually used.

Using this general detection formulation, we will now describe, and compare analytically, two different methods of detection capability estimation:

- estimation from actual detection data (the so-called direct method);
- estimation from noise.

To each of these methods two different approaches may be taken:

- make no statistical assumptions;
- assume a Gaussian probability of detection.

Each method's general discussion is followed by a description of the two approaches. The conditions and restrictions for each method and approach are specified and the expected differences in results from either method or approach are stated.

1. The Direct Method

a. General

The direct method of detection capability estimation computes the percentage of detections out of an ensemble of events in a certain region and of a certain reference magnitude. This percentage represents the probability that an event of a given reference magnitude can be detected. By expressing the percentage as a function of reference magnitude one obtains the detectability curve, and the 50%, 90%, etc., detection thresholds may be determined from these data. The method requires sufficient statistical populations of events of the same reference magnitude in a certain region, over a wide range of reference magnitudes. Surface-wave detection is usually based on SNR, dispersion and expected arrival time.

b. The direct method without making statistical assumptions

Applied in the manner described above, the direct method does not require any statistical model or assumptions, and, in principle, the range of relative epicentral distances, Δ/Δ_0 , is not restricted. A wide range, for instance established by events occurring over a large region at a relatively short distance from the station, probably would make the detectability results less meaningful, however, in the sense that nearby and far away events with the same reference magnitude appear to have the same probability of detection. On the other hand, to meet the requirement of a sufficiently large population of events one may be forced to estimate the detection capability from events occurring over a relatively large region.

c. The maximum likelihood approach to the direct method

An alternative manner of establishing direct estimates is the maximum likelihood approach (Ringdal, 1974). This approach fits a Gaussian probability of detection curve through the detection percentage points obtained by the direct method, yielding the mean and standard deviation of some threshold

magnitude determined by the seismic noise level and the detection algorithm characteristics. The probability of detecting an event of given reference magnitude is then given by the probability that the event station magnitude is greater than the threshold magnitude, given the event reference magnitude.

This is similar to the expression (II-13) where the reference magnitude has been translated into the station magnitude value and the bias term b . The threshold magnitude in the maximum likelihood method then is the same as the noise magnitude plus the period dependent difference term, $M_N + d(T)$, in Equation (II-13).

The assumption that the probability of detection is a Gaussian distribution function is approximately valid if the range of the relative epicentral distances, Δ/Δ_0 , is small, e. g., for

$$0.7 < \Delta/\Delta_0 < 1.6 \quad (\text{II-26})$$

$$|\log \Delta/\Delta_0| < 0.2 \quad (\text{II-27})$$

which may establish an approximately log-normal distribution of Δ . For example, for a region with $\Delta_0 = 50^\circ$, Δ should be within the range 35° to 80° to meet the above constraints.

Although the estimation model in this approach is based on the signal-to-noise ratio detection criterion, it also depends on the dispersion and arrival time criteria since the method uses actual detection data determined with the aid of these criteria.

2. Estimating Detection Capabilities From Noise

a. General

In Subsection II-B we found, according to Equation (II-14) that the probability of detecting an event of given reference magnitude x equals

the probability that the r. v. $M = M_N + d(T) - b$ is less than or equal to x , where M_N (Equation II-10) is the so-called noise magnitude made up by logarithmic terms of the maximum peak-to-peak noise amplitude at the seismometer output, the event signal period T , the instrument response $G(T)$ for that period, the event epicentral distance Δ , and a certain SNR margin. The term $d(T)$ is the empirical difference between T -sec and 20-sec magnitudes, and b is the station magnitude variation about the reference magnitude.

The detection capability of a seismic station, therefore, can be determined immediately if all statistics of the above parameters are known. Thus, we collect noise samples to establish the distribution of the logarithm of the maximum peak-to-peak noise amplitude at the seismometer output. From existing data we determine the distribution of the periods of the maximum event signal amplitudes. With the known instrument response $G(T)$, and the known 20-sec minus T -sec magnitude difference $d(T)$, we then have available the distributions of $\log T$, $\log G(T)$, and $d(T)$, respectively. We then study the seismicity in the area of interest to obtain the distribution of the logarithm of epicentral distances between source and station, $\log \Delta$, and may determine the correlation between the parameters T and Δ . The distribution of the station magnitude variation about the reference magnitude, finally, also is obtained from previous data and its correlation with the parameters T and Δ may be studied.

The above procedure can be applied to existing stations as well as to new or proposed sites. In the latter case of course, a seismometer must be installed to measure the noise amplitudes. Also, the new station's signal period distribution may differ from those of other stations, due to the source radiation pattern and differences in propagation paths. Furthermore, the new station's magnitude variation with respect to the reference magnitude cannot be established, but may be statistically similar to those of other

stations. The instrument response of the new station must be known so that, based on the signal period distributions of other stations, an indication about the $\log G(T)$ distribution may be obtained.

As pointed out in the introduction the method of estimating detection capabilities from noise is fast compared with the direct method, since we only have to collect noise data rather than event signal data. With a well functioning instrument, and depending on the time lapse between the seismicity sensed by the instrument, one-hour noise samples can in general be obtained at least once a day. The empirical evaluation of this method described in Section III will show how many samples are required to obtain an accurate estimate. It is expected that a population of approximately 30 valid noise samples would suffice. This would mean that a station's detection capability could be determined within one to two months depending on the quality of the data. Estimates may have to be adjusted for seasonal noise differences, and for station magnitude bias.

b. Estimation from noise without making statistical assumptions

Up to this point we have not yet made or used any statistical assumptions; and in principle detection capabilities may be estimated by empirically establishing the probability distribution function, $F_M(x)$, of the r.v. M by combining the event signal period, distance, and station bias statistics with the (independent) noise amplitude statistics. The 50%, 90%, etc., detection thresholds then are found from the relationships $F_M(x_{50}) = 0.50$, $F_M(x_{90}) = 0.90$, etc. Since, as far as SNR detection is concerned, the above described method follows exactly the detection mechanism used to obtain the detection data for the direct method, the detection capability estimates from either method should differ very little. The only differences lie in the incorporation of dispersion and arrival time in the detection criterion used in actual detection, and in possibly insufficient populations of events of the same magnitude to determine the detection percentages in the direct method.

c. Estimation from noise assuming a Gaussian detection probability

If the distribution of the r.v. M is an even function the 50% detection threshold is found by averaging the r.v. M , or averaging its constituents and adding the averages. If the r.v. M is Gaussian we can also find the other thresholds in closed form; they are given by the error function. The 90% threshold, for instance, is given by Equation (II-25). Thus, assuming a Gaussian distribution function and determining the detection thresholds from the error function is essentially little different from the maximum likelihood approach to the direct method of estimating detection capabilities. In this case the estimate differences also stem from incorporating dispersion and arrival time in the detection criterion and from the fact that in that method the μ and σ values are obtained from a curve fitting procedure rather than from explicit calculations.

F. THE USE OF RMS VALUES IN ESTIMATING DETECTION CAPABILITIES FROM NOISE

The introduction of this report mentioned Lambert's suggestion that RMS values rather than maximum amplitudes might be used to estimate a station's detection capability from noise. In this subsection we will investigate the validity of that procedure.

Lambert et al. (1973) observed that at the seismometer output, the maximum-noise-amplitude-over-RMS ratio was relatively constant. We may change the expression for the noise magnitude (II-10) into:

$$M_N = \log \text{RMS}_{\text{out}} + \log \frac{A_{N_{\text{out}}}}{\text{RMS}_{\text{out}}} - \log T \cdot G(T) + \log \Lambda + C. \quad (\text{II-28})$$

If, for the case of an even detection probability density function, these terms are averaged to obtain the 50% detection threshold, we may make the substitution:

$$\left[\log \frac{A_{N_{out}}}{RMS_{out}} \right] \approx \log \left[\frac{A_{N_{out}}}{RMS_{out}} \right] \quad (11-29)$$

since the ratio varies little (Appendix C). Thus, if the average value of $A_{N_{out}}/RMS_{out}$ is known the 50% detection threshold value is determined by the RMS noise levels:

$$x_{50} = \bar{M}_N + \bar{d}(T) - \bar{b} \quad (11-30)$$

where

$$\bar{M}_N = \overline{\log RMS_{out}} + \log \frac{\overline{A_{N_{out}}}}{\overline{RMS_{out}}} - \overline{\log T \cdot G(T)} + \overline{\log \Lambda} + C. \quad (11-31)$$

This method may be convenient if for a given station RMS noise values rather than maximum noise amplitudes at the seismometer output are available, and the amplitude-over-RMS ratio is known. Estimating the 90%, etc., thresholds is more complicated since it requires calculating the variance and covariance of the first two terms in Equation (11-31).

G. ESTIMATING THE 50% DETECTION THRESHOLD FOR AN EVEN DETECTION PROBABILITY DENSITY FUNCTION

In this subsection we further consider the process of estimating detection capabilities from noise. In particular, we focus on the averaging process involved in estimating the 50% detection threshold from noise in the case that the detection probability density function is even about its mean. For an even distribution of the r. v. $M = M_N + d(T) - b$ the 50% detection threshold is given by the mean of M , Equation (11-24):

$$x_{50} = \mu_M = \bar{M}_N + \bar{d}(T) - \bar{b}. \quad (11-32)$$

Substituting the expression for the noise magnitude M_N , Equation (11-10), and following Equation (11-16) we obtain:

$$x_{50} = \overline{\log A_{N_{out}}} - \overline{\log T} - \overline{\log G(T)} + \overline{\log \Lambda} \\ + \overline{d(T)} - \overline{b} + C. \quad (II-33)$$

Thus, we must first compute the logarithm of the parameters $A_{N_{out}}$, T , $G(T)$, and Λ , then average the logarithms for each of these parameters, and algebraically add these averages to $\overline{d(T)}$, \overline{b} , and C . Taking the average of the logarithms is equivalent to taking the logarithm of the geometric mean. In Appendix C it is shown that for a population with a small σ/μ ratio, the geometric mean approximately equals the arithmetic mean. The geometric mean has the characteristic that it weighs extreme values less heavily.

According to the above, the terms $\overline{\log T}$ and $\overline{\log G(T)}$ may be replaced by $\log T_0$ and $\log G(T_0)$, respectively, where T_0 is the geometric mean of the signal period T . Since, furthermore, $d(T)$ is approximately linear with $\log T$, the term $\overline{d(T)}$ may similarly be replaced by $d(T_0)$. Alternatively, we may express the 50% threshold as a function of T :

$$x_{50}(T) = \overline{\log A_{N_{out}}} - \log T - \log G(T) + \overline{\log \Lambda} \\ + d(T) - \overline{b} + C \quad (II-34)$$

where now the period T is a running parameter rather than a mean value.

Similar considerations hold for the epicentral distance term. According to Equation (II-33) we must take the logarithm of its geometric mean, Λ_0 :

$$\overline{\log \Lambda} = \log \Lambda_0. \quad (II-35)$$

Since the $\log \Lambda$ distribution is in general not normal we want to either confine the Λ/Λ_0 range, or make Λ a running parameter:

$$x_{50}(\Lambda) = \overline{\log A_{N_{out}}} - \overline{\log T \cdot G(T)} + \log \Lambda \\ + \overline{d(T)} - \overline{b} + C . \quad (II-36)$$

Expressing the detection thresholds as a function of T and Λ creates families of detection capability curves, each curve representing a certain period and distance.

H. SUMMARY OF THE ANALYTICAL RESULTS

Based on the SNR detection criterion, the foregoing subsections developed the probability of detecting a seismic event of a given reference magnitude x into the probability expression:

$$P(\det \bar{M}_s(20) = x) = P(M \leq x) = F_M(x) , \quad (II-37)$$

where:

$\bar{M}_s(20)$ is the event 20-sec reference magnitude;

$M = M_N + d(T) - b$;

M_N is the "noise magnitude" ;

T is the period of the maximum event signal amplitude at the seismometer output ;

$d(T)$ is the T -sec magnitude correction with respect to the 20-sec magnitude ;

b is the station magnitude variation about the reference magnitude ;

$F_M(x)$ is the probability distribution function of M .

The "noise magnitude" is given by:

$$M_N = \log A_{N_{out}} - \log T \cdot G(T) + \log \Lambda + C , \quad (II-38)$$

where:

$A_{N_{out}}$ is the maximum peak-to-peak noise amplitude at the seismometer output ;

$G(T)$ is the instrument response for the signal period T ;

Δ is the epicentral distance between the event source and the station concerned, in degrees ;

$C = 1.12 + \text{detection criterion margin.}$

If the statistics of all terms constituting the r.v. M are known its probability distribution function, $F_M(x)$, is also known, and the 50%, 90%, etc., detection thresholds, x_{50} , x_{90} , etc., may be found from:

$$F_M(x_{50}) = 0.50 , \quad (II-39)$$

$$F_M(x_{90}) = 0.90 . \quad (II-40)$$

If $F_M(x)$ is an even function about the mean of M the 50% detection threshold equals this mean:

$$x_{50} = \mu_M . \quad (II-41)$$

If, moreover, the r.v. M is Gaussian, the detection thresholds are found from the argument x of the Gaussian probability distribution function

$$F_M(x) = 0.5 + \text{erf} \frac{x - \mu_M}{\sigma_M} , \quad (II-42)$$

where σ_M is the standard deviation of the r.v. M . In particular, the 50% and 90% detection thresholds are then given by

$$x_{50} = \mu_M \quad (II-43)$$

and

$$x_{90} = \mu_M + 1.28 \sigma_M . \quad (II-44)$$

It was shown from available statistics that the r.v. M is approximately a linear combination of approximately normally distributed variables or constants, if the relative epicentral distance range, Δ/Δ_0 , is kept small. Therefore, also the r.v. M itself is approximately normally distributed under the above mentioned distance constraint. If, moreover, the parameters T , Δ , and b are uncorrelated, the standard deviation of M can be computed from the standard deviation concerning each parameter involved:

$$\sigma_M^2 = \sigma_{\log A_{N_{out}}}^2 + (\beta - \alpha - 1) \sigma_{\log T}^2 + \sigma_{\log \Delta}^2 \quad (II-45)$$

where

β = rate of change of $d(T)$ with $\log T$, $20 \leq T \leq 30$;

α = rate of change of $\log G(T)$ with $\log T$, $20 \leq T \leq 30$.

From available data for well functioning VLPE stations we found

$$0.39 < \sigma_M < 0.55 . \quad (II-46)$$

The above formulation is a model of the SNR detection mechanism used by the analyst. This model does not take into account the criteria of dispersion and arrival time. Based on this formulation two methods of estimating detection capabilities and two approaches to each method were discussed: the direct method and its maximum likelihood approach; and the method of estimating the detection capability from noise, with and without the assumption of a Gaussian probability of detection. It was found that there should be little difference between the direct estimates and the estimates obtained from noise. Estimating detection capabilities from noise is a relatively fast process, but it requires statistical information about the signal periods,

the epicentral distances and the station magnitude bias, which information may be available from previously processed events. Accurate estimates should be obtainable from as few as 30 valid noise samples. This method may use either maximum noise amplitudes or RMS noise levels; either one must be measured at the seismometer output. In the latter case the maximum-amplitude-over-RMS noise ratio at the seismometer output must be known.

SECTION III

EMPIRICAL EVALUATION

The foregoing theory was tested with data available from previous VLPE processing. Noise samples taken by six VLPE stations during the period November 1972 through January 1973 were edited for extreme values and used in estimating the detection capabilities of the stations concerned. The results were compared with estimates obtained by the maximum likelihood approach to the direct method (Lambert et al., 1973). The following subsections describe the available data, the noise data processing, and the procedure of estimating detection capabilities from noise. The empirical results, presented in tabular form, are discussed and compared with the conclusions drawn from the analysis in the previous section.

The locations of the stations of the VLPE network are given in Table III-1 and Figure III-1. The stations TLO, OGD, and FBK were not in operation during the above mentioned period, and the data from EIL and MAT were considered unreliable.

A. AVAILABLE DATA

In the following we will briefly describe the available noise data and maximum likelihood detection capability estimates used to perform the empirical evaluation.

1. Noise Data

Ensembles of noise samples collected during the period November 1972 through January 1973 at each of the VLPE stations CTA, CHG, KON, KIP, ALQ, and ZLP were used to estimate detection capabilities from

TABLE III-1
VERY LONG PERIOD EXPERIMENT (VLPE)
STATIONS AND LOCATIONS

Station	Designator	Latitude	Longitude
Charters Towers, Australia	CTA	20.09S	146.26E
Chiang Mai, Thailand	CHG	18.79N	98.98E
Fairbanks, Alaska	FBK	64.90N	148.01 W
Toledo, Spain	TLO	39.86N	4.02 W
Eilat, Israel	EIL	29.55N	34.95E
Kongsberg, Norway	KON	59.65N	9.59E
Ogdensburg, New Jersey	OGD	41.07N	74.62 W
Kipapa, Hawaii	KIP	21.42N	158.02 W
Albuquerque, New Mexico	ALQ	34.94N	106.46 W
La Paz, Bolivia	ZLP	16.50S	68.13 W
Matsushiro, Japan	MAT	36.54N	138.12E

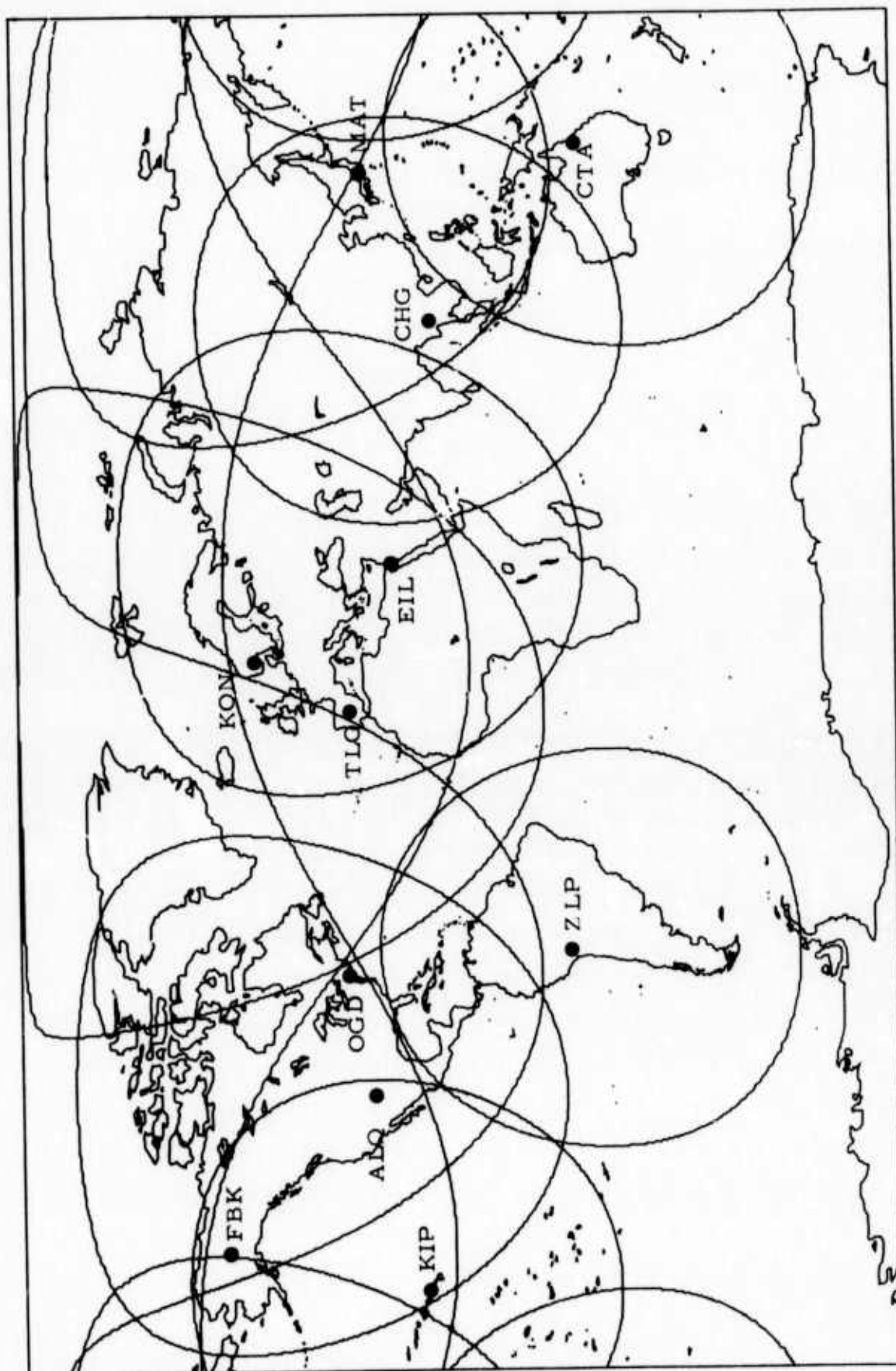


FIGURE III-1
MAP OF VLPE STATIONS AND CIRCLES AROUND EACH SITE WITH RADIUS OF 50° DISTANCE

noise. The noise samples consist of digitized seismograms taken over approximately one-hour periods when, according to PDE, NORSAR, and LASA listings, no seismic activity was present. The digital data expressed in computer count (c. c.) units were bandpassed from 0.0 to 0.14 Hz via Fourier transforms, and normalized such that amplitudes with a 40-sec period represent true ground motion in millimicrons. The relationship between input (true ground motion) and output amplitudes as a function of period then is given by the station instrument response $G(T)$ with

$$G(40) = 1.0 , \quad (\text{III-1})$$

and the response at all other periods is scaled relative to $G(40)$. A typical instrument response curve is given in Appendix A (Figure A-9); the instrument response curves for all stations of the VLPE network are given by Lambert et al. (1973). Table A-1 lists the 20-sec and 30-sec instrument responses for the period that the data were collected. Further details on collecting and processing VLPE noise data are given by Prah1 (1974).

2. Maximum Likelihood Detection Capability Estimates

The detection capability estimates obtained from these noise data was to be compared with estimates found by means of the maximum likelihood approach to the direct method applied to actual detection versus non-detection of events which occurred during the periods January 1, 1972 through March 20, 1972; June 1, 1972 through August 31, 1972; and November 1, 1972 through December 31, 1972. The maximum likelihood detection threshold estimation was expressed in terms of reported bodywave magnitudes, and the bodywave magnitude 50% detection thresholds were subsequently converted to 20-sec surface wave magnitude 50% detection thresholds using the relationship

$$M_s(20) = 1.20 m_b - 1.74 . \quad (\text{III-2})$$

The same relationship is used to establish a reference surface wave magnitude, Equation (II-4). This conversion formula was established by least mean square fitting of available m_b and M_s data by Lambert and Becker (1973). Furthermore, the maximum likelihood 90% detection thresholds were determined from

$$x_{90} = \mu + 1.28\sigma = x_{50} + 1.28\sigma \quad (\text{III-3})$$

in which a σ -value of 0.4 was used for all stations. According to the discussion in Subsection II-C this constant σ -value used for all stations may have been a low estimate for some stations, since for our Gaussian detection probability model we found a range

$$0.39 < \sigma_M < 0.55 \quad (\text{III-4})$$

depending on the station noise amplitude distribution, the propagation path, the instrument response, and the range of epicentral distances.

B. NOISE DATA PROCESSING

The noise data described in the previous subsection needed to be bandpassed to agree with the event signal processing band, and edited to remove samples taken during station malfunction or station calibration, and samples containing spikes and unreported signals. The best way of editing the data is visual inspection of the wide-band noise traces. Even then, in some cases, some ambiguity remains as to whether a sample contains high level noise or one of the anomalies mentioned above. Furthermore, CALCOMP plotting and the visual inspection of the plotted traces for all samples (some 600 traces) was considered too time consuming (especially since an interactive display capability was being developed for the PDP-15 computer at the Seismic Data Analysis Center (SDAC) and would be available for spot-checking data quality). Therefore, the following automatic, but knowingly

crude, method was applied while also preparing the data for detection threshold estimation. To facilitate reference to the computer-generated Tables III-3 through III-8, and E-1 through E-6, the variables (AMP, RMS, AMP/RMS, etc.) presented in those tables are accordingly given in capital letters in the following text.

The available (wide-band) noise data were subjected to a 17-44 sec bandpass filter. This passband was also used in standard processing of VLPE event signals. From the bandpassed noise data, the maximum peak-to-peak amplitude (AMP) and RMS values were determined. The peak-to-peak amplitudes were found by a search algorithm which also yields the period (AMP PER) of the maximum noise amplitude. The RMS value is computed in the time domain as

$$\text{RMS} = \left[\frac{1}{N} \sum_{i=1}^N n^2(i) \right]^{1/2}, \quad (\text{III-5})$$

where

$n(i)$ is the i^{th} data point noise value in $m\mu$;

N is the total number of data points in the time trace.

Next, the values AMP/RMS, LOG AMP, LOG RMS, LOG RATIO (= log AMP/RMS) were computed for each sample. The mean and standard deviation of the values AMP, RMS, AMP/RMS, LOG AMP, LOG RMS, and LOG RATIO were determined according to

$$\text{MEAN} = \frac{1}{N} \sum_{i=1}^N d(i) \quad (\text{III-6})$$

and

$$\text{STD.DEV.} = \left[\frac{1}{N-1} \left(\sum_{i=1}^N d^2(i) - N \cdot \text{MEAN}^2 \right) \right]^{1/2} \quad (\text{III-7})$$

where $d(i)$ is the i^{th} data point of the above parameters.

To show the difference between mean-of-the-logarithms and logarithm-of-the-mean, (Section II-G and Appendix C), the value LOG MEAN was computed for the means of AMP, RMS, and AMP/RMS.

Editing of the noise data was then performed by rejecting samples for which at least one of the values AMP/RMS, LOG AMP, LOG RMS, or LOG RATIO fell outside an interval of NSD times their standard deviation about the mean. The number NSD can be any number; in the processing presented here NSD = 2 was selected. This editing process was continued until all data fell within the desired interval. Final editing results are displayed in Appendix E for all stations involved.

This editing process may not be stable. If the data contains many high extremes it may force out the low values and bias the population high. In general, noise data do not contain anomalously low values, but the population may still be biased low if the rejected high values were established by actual seismic noise. If the distribution of values is approximately normal, however, the above criterion is fairly reasonable. Intentionally, therefore, we do not edit the maximum amplitudes or RMS values, which are not normally distributed, but rather we edit their logarithms which approximate normal random variables. In addition, the supposedly near-constant values AMP/RMS were edited in the same manner, which should serve especially to remove spikes and possibly d.c. levels.

After the processing was completed the PDP-15 interactive display at the SDAC was used for a quick check on the above procedure by means of a simple visual inspection of the wide band noise traces. As expected, the above procedure proved by no means perfect, but the remaining anomalous data and the rejection of some presumably good data were judged not to significantly distort the population of valid noise samples. The resulting data displayed in Appendix E, at least, seem plausible. The ZLP waveforms

appeared somewhat distorted. The VLPE noise study by Prahl (1974) discusses the general editing process in more detail.

C. ESTIMATING DETECTION THRESHOLDS FROM NOISE

The detection capability of a seismic station may be estimated by determining the detection thresholds as indicated in Section II. As shown, we need not necessarily assume a statistical model as long as we are able to determine the detection probability distribution function, $F_M(x)$, otherwise. Since we want to compare our results with estimates from the maximum likelihood direct method, which assumes a Gaussian distribution, we also assumed $F_M(x)$ to be normal. As pointed out in the analysis, this assumption is approximately valid if the range of relative epicentral distances, Δ/Δ_0 is kept small. The available maximum likelihood estimates, however, were determined from events spread over relatively large regions, and for most stations the log Δ distribution is not normal, not even where the event populations were divided into subsets of $\Delta < 50^\circ$ and $\Delta \geq 50^\circ$, as seen from Appendix A. Since both methods are based on the same detection mechanism formulated in Section II, they will make approximately the same error in making the assumption of normality for the probability of detection, and should still yield similar results.

As was shown further in Section II, the 50% detection threshold is determined from the average of the r. v. M . This means that we must use the arithmetic mean of the logarithms, or equivalently, the logarithm of the geometric mean of the noise amplitude, AMP, the signal period T , and the epicentral distance, Δ . We furthermore need the value of the station magnitude bias \bar{b} , and an estimate of the s. d. for the parameters $\log \text{AMP}$, $\log T$, $\log \Delta$, and for the station M_s variation or bias term b . Furthermore, the instrument response $G(T)$, and the constant $C = 1.12 + \text{detection SNR margin}$ are required.

The above considerations lead to the following procedure:

- Determine the mean and s. d. of the logarithm of the maximum peak-to-peak seismometer output noise amplitudes;
 - MEAN LOG AMP, LOG AMP S. D.
- Determine from previously processed events in the region of interest, for the parameters T , Λ , and b ;
 - the logarithmic distribution of T and Λ ;
 - the geometric means T_0 , Λ_0 ;
 - the s. d. of the logarithms: $\sigma_{\log T}$, $\sigma_{\log \Lambda}$;
 - station M_s bias \bar{b} and s. d. σ_b .
- Input to the detection capability estimator the values
 - MEAN LOG AMP, LOG AMP S. D. ;
 - T_0 , $G(T_0)$; other T , $G(T)$ values;
 - $G(20)$, $G(30)$;
 - Λ_0 , other Λ 's ;
 - \bar{b} (BIAS) and σ_b (BIAS S. D.) ;
 - $d(T) = MS(20) - MS(PER)$.

The detection capability estimation algorithm then computes:

- The 50% detection threshold:
 - $x_{50} = \mu_M = \text{MEAN LOG AMP} - \log T_0 \cdot G(T_0)$
 $+ \log \Lambda_0 + d(T_0) - \bar{b} + C$; (III-8)
- The standard deviations:
 - SIG PER EFFECTS S. D. = $\sigma_M^T = (\beta - \alpha - 1) \sigma_{\log T}$; (III-9)
 - TOTAL S. D. = $\sigma_M = \left[(\text{LOG AMP S. D.})^2 + (\text{SIG PER EFFECTS S. D.})^2 + (\text{DIST S. D.})^2 + (\text{BIAS S. D.})^2 \right]^{1/2}$, (III-10)

where

$$\text{DIST S. D.} = \sigma_{\log \Delta}.$$

- The 90% detection threshold:

$$x_{90} = \mu_M + 1.28 \sigma_M. \quad (\text{III-11})$$

The value MEAN LOG AMP may be replaced by (MEAN LOG RMS + LOG MEAN RATIO) if the use of RMS values rather than peak-to-peak amplitudes is preferred, and the mean ratio is known.

Since most maximum signal amplitudes were found to have a 30-sec period, Figure A-7, and since detection thresholds are given as 20-sec magnitudes, we computed the thresholds for 20-sec and 30-sec signal periods, rather than for the signal period geometric mean. The results then reflect the range of detection capability estimates as a function of signal period. Furthermore, where applicable, we computed the thresholds for the geometric mean of the Δ -values for the total region, the $\Delta < 50^\circ$ region, and the $\Delta \geq 50^\circ$ region. Finally, we assumed the stations to be unbiased in magnitude (STA MS BIAS = 0.0) with a s. d. of 0.35 (BIAS S. D. = 0.35).

D. PRESENTATION OF THE RESULTS

The results of the editing and averaging process described in Subsection III-B are given in the tables of Appendix E. Some of the key values from these tables are combined in an overview in Table III-2 to evaluate the logarithm-of-the-mean versus the mean-of-the-logarithm operation on noise amplitudes, and the consistency of AMP/RMS ratios. The threshold estimates and related values as described in Subsection III-C are listed in Tables III-3 through III-8. The 50% and 90% detection threshold estimates from noise, M50 and M90, are given for the case that noise AMPLITUDE values were used as well as for the case of threshold estimation from RMS noise

TABLE III-2
MEAN-OF-THE-LOGARITHM VERSUS LOGARITHM-OF-THE-MEAN
OPERATION ON NOISE AMPLITUDES; AMP/RMS
CONSISTENCY EVALUATION

Station	μ	σ	$0.215(\sigma/\mu)^2$	$\log \mu$	μ_{\log}	Difference	AMP/RMS	σ_{Ratio}
CTA	58.47	30.51	0.06	1.77	1.72	0.05	8.61	2.14
CHG	26.59	5.76	0.01	1.42	1.42	0.00	7.34	1.20
KON	43.98	11.21	0.01	1.64	1.63	0.01	6.55	0.58
KIP	44.74	9.05	0.01	1.65	1.64	0.01	6.73	0.80
ALQ	44.42	8.76	0.01	1.65	1.64	0.01	6.26	0.36
ZLP	32.11	9.12	0.02	1.51	1.49	0.02	6.60	0.68

TABLE III-3
DETECTION CAPABILITY ESTIMATION FROM NOISE: STATION CTA

[illegible]

TABLE III-4

31 NCIS SAMPLES

TABLE III-5
DETECTION CAPABILITY ESTIMATION FROM NOISE: STATION KON

STATION		6 KON	DAYS	72306-73	30	32 NOISE SAMPLES									
LOG AMP		S.D.=0.11		SIG PPR EFFECTS		S.D.=0.10									
STA MS		BIAS=0.0		BIAS S.D.=0.35											
EPIC DIST	DIST S.D.	SIG	PER	INST RESP	MS(20)- MS(PER)	TOTAL S.D.	AMPLITUDE		MT	RMS		M90	DIRECT		
							MT	M50		M50	M90		M50	M90	
47.0	0.2	20	0.460	0.0	0.43	0.43	3.61	3.61	4.16	3.61	3.61	4.16	3.52	4.02	
47.0	0.2	30	0.920	0.30	0.43	0.43	3.13	3.43	3.98	3.13	3.43	3.98	3.52	4.02	
32.1	0.2	20	0.460	0.0	0.43	0.43	3.44	3.44	3.99	3.44	3.44	3.99	3.37	3.87	
32.1	0.2	30	0.920	0.30	0.43	0.43	2.97	3.27	3.82	2.97	3.27	3.82	3.37	3.87	
65.1	0.1	20	0.460	0.0	0.39	0.39	3.75	3.75	4.25	3.75	3.75	4.25	3.60	4.10	
65.1	0.1	30	0.920	0.30	0.39	0.39	3.27	3.57	4.08	3.27	3.57	4.08	3.60	4.10	

TABLE III-6
DETECTION CAPABILITY ESTIMATION FROM NOISE: STATION KIP

STATION R KIP DAYS 72306-73 31 48 NOISE SAMPLES							
LOG AMP S.D.=0.09		SIG PER EFFECTS S.D.=0.20					
STA MS BIAS=0.0		BIAS S.D.=0.35					
EPIC DIST S.D.	SIG PER RESP	INST MS(20)- MS(PER)	TOTAL S.D.	AMPLITUDE		RMS	
				MT	M50	MT	M50
				M90	M90	M90	M90
74.2	0.2	20 0.510	0.0	3.77	3.77	3.78	3.78
74.2	0.2	30 0.990	0.12	3.31	3.43	3.31	3.43
46.0	0.0	20 0.510	0.0	3.57	3.57	3.57	3.57
46.0	0.0	30 0.990	0.12	3.10	3.22	3.10	3.22
100.5	0.1	20 0.510	0.0	3.91	3.91	3.91	3.91
100.5	0.1	30 0.990	0.12	3.44	3.56	3.44	3.56

TABLE III-7

30 NOISE SAMPLES

TABLE III-8

DETECTION CAPABILITY ESTIMATION FROM NOISE: STATION ZLP

STATION 10 ZIP DAYS 72320-73 24 21 NOISE SAMPLES									
LOG AMP S.D.=0.12 SIG PER EFFECTS S.D.=0.15									
STA MS BYAS=0.0 PIAS S.D.=0.35									
EPIC	DIST	SIG	INST	MS(20)-	TOTAL	AMPLITUDE		RMS	DIRECT
DIST	S.D.	PER	PESP	MS(PER)	S.D.	MT	W50	W90	W90
120.3	0.1	20	0.365	0.0	0.41	4.01	4.01	4.01	4.12
120.3	0.1	30	0.890	0.30	0.41	3.45	3.75	3.75	4.12

levels. The value MT represents the noise magnitude part of the 50% detection threshold; it excludes $d(T)$ and the bias term b :

$$MT = M_N = \text{MEAN LOG AMP} - \text{LOG } T G(T) \\ + \text{LOG } \Delta_o + C. \quad (\text{III-12})$$

Furthermore, the tables give the DIRECT 50% and 90% detection threshold estimates as determined by the maximum likelihood direct method from VLPE data. Table III-9 gives an overview of 50% detection threshold differences and related values.

E. DISCUSSION

We will first discuss the data presented in Appendix E and in Table III-2. The log amplitude distributions of the bandpassed and edited noise data are given in Appendix A. They are approximately normally distributed and their standard deviations range from 0.1 to 0.2. Furthermore, we notice that for all parameters

$$|\text{LOG MEAN} - \text{MEAN LOG}| \approx 0.215 (\sigma/\mu)^2 \quad (\text{III-13})$$

as predicted in Appendix C. For most stations this difference is less than 0.02 indicating that one could take the logarithm of the arithmetic rather than the geometric mean without introducing a significant error. This also holds for the AMP/RMS ratio, which, however, varies more than expected at the station CTA. This is the only station with a definitely non-log-normal noise amplitude distribution. Thus, we may estimate detection thresholds from RMS noise levels rather than from noise peak-to-peak amplitudes if preferred, using a fixed AMP/RMS ratio. This ratio only seems to be consistent among the stations KON, KIP, ALQ, and ZLP, however, and the value of this ratio must apparently be established separately for each station, so that this method offers little advantage, especially when evaluating a new site.

TABLE III-9
DETECTION CAPABILITY ESTIMATION FROM NOISE VERSUS
MAXIMUM LIKELIHOOD DIRECT ESTIMATION

Station	Δ_0	From Noise 72306 - 73031				Direct		Difference	
		Number of Samples	σ log A	50% Threshold		Number of Events	50% Threshold	20 sec	30 sec
				20 sec	30 sec				
CTA	82.5	26	0.21	4.21	3.64	313	3.73	0.48	-0.09
CHG	43.8	31	0.09	3.65	3.33	387	3.48	0.17	-0.15
	33.0			3.53	3.21	208	3.28	0.25	-0.07
	61.3			3.80	3.48	179	3.66	0.14	-0.18
KON	47.0	32	0.11	3.61	3.43	485	3.52	0.09	-0.09
	32.1			3.44	3.27	228	3.37	0.07	-0.10
	65.1			3.75	3.57	257	3.60	0.15	-0.03
KIP	74.2	48	0.09	3.77	3.43	542	3.62	0.15	-0.19
	46.0			3.57	3.22	203	3.42	0.15	-0.20
	100.0			3.91	3.56	339	3.77	0.14	-0.21
ALQ	85.1	30	0.09	3.85	3.67	332	3.94	-0.09	-0.28
ZLP	128.5	21	0.12	4.01	3.75	106	4.12	-0.11	-0.35

Although the noise amplitude periods (AMP PER) are not required in the calculation of detection thresholds they show the interesting fact that they are in general in the 20 to 32 sec range; their distributions are given in Appendix B.

We now turn to the results presented in Tables III-3 through III-9. The standard deviations of the combined signal period effects were already given in Table II-1; together with the log amplitude s. d., the log distance s. d., and the bias s. d., they yield a total σ_M which ranges from 0.39 to 0.53, in accordance with Equation (II-18).

The threshold difference due to using either 20-sec or 30-sec signal periods according to Equation (A-6) is approximately:

$$\begin{aligned} M50(20) - M50(30) &\approx (\beta - \alpha - 1)(\log 20 - \log 30) \\ &= -0.18 (\beta - \alpha - 1) . \end{aligned} \tag{III-14}$$

This difference is greater for stations with signals received over mixed continental-oceanic propagation paths and for stations with a large $G(30)/G(20)$ instrument response ratio. Over the six stations used in the evaluation, the difference ranges from 0.18 (KON) to 0.58 (CTA) as is also reflected by Table A-1. Since the station magnitude bias is assumed zero, the detection threshold for a 20-sec signal period is given by the noise threshold magnitude MT ; for 30-sec signal period detections we must add the period dependent magnitude difference terms given by Equation (III-14).

According to the overview in Table III-9, the 50% detection thresholds show excellent agreement between the estimates obtained from noise samples and the maximum likelihood direct estimates. For three of the six stations (CHG, KON, KIP) the direct estimate lies about midway between the 20- and 30-sec estimates from noise. According to the confidence level in the maximum likelihood detectability curves (Lambert et al., 1973)

the direct estimates for these stations seem reliable. This may indicate that the geometric mean of the periods of maximum signal amplitude for those stations is approximately 25 seconds.

The CTA estimate from noise seems somewhat high, although the direct estimate still lies between the 20- and 30-sec signal period noise thresholds. This may be related to the strong total effect of the signal period due to the combination of a steep instrument response and the fact that the station receives its signals over mixed continental-oceanic paths. Also, the noise amplitude distribution is not log-normal, and the log amplitude s.d. is higher than those of CHG, KON, KIP, ALQ, and ZLP. The direct estimates for this station may be less reliable due to a relatively low number of higher magnitude events; the confidence limits seem somewhat wide.

The stations ALQ and ZLP show a 0.1 to 0.3 lower detection threshold estimated from noise as compared with the direct estimate. ALQ noise traces looked good on the interactive display; its log amplitude distribution is close to normal, and its log amplitude s.d. and the combined signal period effects s.d. are small. The ALQ direct estimate may be less reliable than that of the other stations, since the population of processed seismic events seems to contain relatively few high magnitude events; the confidence limits are wider than those for CTA. The same situation exists to a greater extent for the station ZLP, for which the overall population of processed events is low. Also, the display of noise traces consistently showed a rather unusual waveform for this station, which may affect the detection capability estimates from noise as well as the direct estimates. Nevertheless, the log amplitude distribution is close to normal and since the waveform shape may not disturb the detection mechanism very much, the estimates from noise may well be reliable.

The 90% detection thresholds are difficult to compare since, if the 50% thresholds agree, the 90% threshold differences depend solely on

the standard deviation, σ , used in $x_{90} = x_{50} + 1.28\sigma$. As discussed earlier, the standard deviation used in maximum likelihood direct estimation obtained via regression on the bodywave to surface-wave magnitude relationship seems low with respect to the s.d. estimated from the statistics of each term involved in estimating detection thresholds from noise. This is reflected in the results given in Tables III-3 through III-8 where the TOTAL S. D. in estimation from noise should be compared with the value $\sigma = 0.4$ used in the direct method.

From the above considerations we may conclude that estimating the detection capability from ambient noise is a valid, fast, and accurate method. A population of approximately 30 valid noise samples already gives reliable estimates. This means that for a well performing station, taking noise samples probably once or twice a day, and accounting for 50% rejection (e.g., KIP), a reliable detection capability estimate can be established within one to two months.

The method also required, however, statistics on the periods of maximum event signal amplitudes, the epicentral distances, and the station magnitude bias. For existing stations these statistics may be obtained from previously processed events. For a new or proposed site the epicentral distances can be calculated from the seismicity in the region of interest, whereas the signal period and magnitude bias statistics can only be estimated, assuming that these statistics will be similar to those of the existing stations. Among the VLPE stations the signal period statistics looked quite similar; there is yet insufficient information about the station magnitude bias statistics. In the evaluation presented here the signal period statistics of the station KIP were considered typical, and were applied to all six stations. Furthermore, a zero magnitude bias with a s.d. of 0.35 was assumed at each station. Evidently, that procedure worked well for all stations used in the evaluation and, unless the physics (path, local structure, instrumentation) of a new station

are definitely different, it is expected that estimating signal period and magnitude bias statistics from the statistics of existing stations will not introduce a significant error in estimating the new station's detection capability from noise.

Finally, it should be noted that the accuracy of the detection thresholds estimated from noise was established with respect to the maximum likelihood estimates. As pointed out previously, both, the maximum likelihood method, and the estimation-from-noise procedure presented here assume a Gaussian model for the probability of detection. A deviation from this model by any of the random variables involved would cause approximately the same error in either method. Thus, although these two methods show excellent agreement, they may both be in error with respect to the true probability of detection.

SECTION IV

SUMMARY

If detection of a seismic event at a given station is based on the surface wave signal-to-noise ratio (SNR) the probability of detecting an event given its 20-sec reference magnitude is given by the probability distribution function of a r.v. M :

$$P(\det \bar{M}_s(20) = x) = P(M \leq x) = F_M(x), \quad (\text{IV-1})$$

where

$\bar{M}_s(20)$ is the 20-sec reference magnitude for the event concerned;

x is the numerical value of the reference magnitude;

$F_M(x)$ is the probability distribution function of the r.v. M .

The r.v. M is described by

$$M = M_N + d(T) - b, \quad (\text{IV-2})$$

where

M_N is the so-called "noise magnitude";

$d(T)$ is the 20-sec minus T -sec magnitude difference;

T is the period of the maximum event signal amplitude at the seismometer output;

b is the station magnitude variation about the reference magnitude.

The "noise magnitude" is given by

$$M_N = \log A_{N_{out}} - \log T \cdot G(T) + \log \Delta + C, \quad (IV-3)$$

where

$A_{N_{out}}$ is the maximum peak-to-peak noise amplitude at the seismometer output;

T is the signal period defined above;

$G(T)$ is the instrument response for the period T , scaled so that a 40-sec output amplitude represents true ground motion in millimicrons, $G(40) = 1$;

Δ is the epicentral distance for the event and station concerned;

$C = 1.12 + \text{a SNR detection margin (e.g., 0.15 for a 3 dB margin.)}$

The term "noise magnitude" is not quite appropriate since it consists of a noise parameter ($A_{N_{out}}$) on the one hand, and event signal parameters (T, Δ) on the other.

The above model of the detection mechanism does not presume any special statistical characteristics, and therefore may serve as a model for any method of estimating a seismic station's detection capability. Based on this model two methods of estimating detection capabilities, and two approaches to each method were discussed:

- Estimation from actual detection percentages
 - not assuming statistical characteristics (the so-called direct method)
 - assuming a Gaussian probability of detection (the maximum likelihood approach to the direct method).

- Estimation from noise
 - not assuming specific statistical characteristics;
 - assuming a Gaussian probability of detection.

Available data indicates that the probability of detecting an event given its magnitude can be considered to be approximately Gaussian, if the relative distance range, Δ/Δ_0 , is kept small (e.g., $\Delta_0 = 50^\circ$, $35^\circ < \Delta < 80^\circ$; $\Delta_0 = 100^\circ$, $70^\circ < \Delta < 160^\circ$).

In the general case the 50%, 90%, etc., detection thresholds (x_{50} , x_{90} , etc.) are found from

$$F_M(x_{50}) = 0.5 \quad (\text{IV-4})$$

and

$$F_M(x_{90}) = 0.9. \quad (\text{IV-5})$$

The Gaussian probability of detection is described by

$$F_M(x) = 0.5 + \operatorname{erf} \frac{x - \mu_M}{\sigma_M} \quad (\text{IV-6})$$

where μ_M and σ_M are the mean and standard deviation, respectively, of the r.v. M . In this case the detection thresholds are determined by μ_M and σ_M :

$$x_{50} = \mu_M \quad (\text{IV-7})$$

and

$$x_{90} = \mu_M + 1.28 \sigma_M. \quad (\text{IV-8})$$

In estimating detection capabilities from noise the statistics of each term constituting the r.v. M , i.e., the statistics of noise amplitude, event signal period, event epicentral distance, and station magnitude bias, are combined to yield the detection probability distribution function $F_M(x)$.

In the general case the entire $F_M(x)$ curve must be established; in the Gaussian case we only need to determine the mean and s.d. of M .

Instrument output RMS noise values may be used instead of maximum noise amplitudes in this method if the average maximum-amplitude-over-RMS ratio is known for the station concerned. This ratio was approximately equal among four out of six VLPE stations.

Based on the analysis of the two methods and their approaches it was concluded that for the non-Gaussian as well as for the Gaussian case there should be little difference between the estimates obtained from actual detection percentages and those obtained from noise samples. This conclusion was supported by an empirical evaluation performed on six stations of the VLPE network, by comparing detection capabilities estimated from noise, assuming a Gaussian detection model, with estimates obtained by the maximum likelihood method. The 50% detection threshold estimates of the two methods agree in general within 0.2 magnitude units. The 90% detection thresholds differ due to a difference in variance between the two methods, since the maximum likelihood variance was obtained through m_b to M_s conversion. The minimum statistical population for estimating detection capabilities from noise appears to be in the order of 30 valid noise samples, and a station's detection capability may be established within one or two months for a well-performing instrument.

This technique, however, requires a prior knowledge of the statistics of the periods of maximum signal amplitudes, the epicentral distances, and the station magnitude bias. For an existing station these statistics may be obtained from previously processed events. For a new or proposed site the distance statistics can be established from the seismicity in the region of interest. The other statistics may be estimated from the event processing performed for other stations of a similar physical configuration. Furthermore, the estimates obtained from noise may require seasonal adjustments.

Finally, it is pointed out that deviation of the statistics concerned from the Gaussian detection model will introduce similar errors in the maximum likelihood estimates and in the estimates obtained from noise. Agreement between these two methods, therefore, does not necessarily guarantee that these estimates are correct with respect to the actual detection capabilities.

Related topics that need further investigation are the distributions and mutual correlations of signal period, epicentral distance, and station surface wave magnitude bias; procedures for the selection of valid noise samples; extension of this study to bodywave magnitudes and the surface-wave horizontal components; and comparison of non-Gaussian detection capability estimation from noise with the non-Gaussian detection estimates of the direct method.

SECTION V

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APPENDICES

APPENDIX A

STATISTICS OF THE RANDOM VARIABLE M

This appendix investigates the statistics of the terms and their combinations contributing to the r. v. M given by Equation (II-16):

$$M = \log A_{N_{out}} - \log T - \log G(T) + \log \Delta + C + d(T) - b \quad (A-1)$$

where

- $A_{N_{out}}$ is the maximum peak-to-peak noise amplitude at the seismometer output;
- T is the period of the maximum event signal amplitude;
- $G(T)$ is the instrument response for the period T ;
- Δ is the event epicentral distance;
- C combines the distance correction term (1.12) and a detection margin term;
- $d(T)$ is the 20-sec minus T -sec magnitude difference;
- b is the station magnitude variation with respect to the reference magnitude.

The following subsections discuss the statistics of the terms: $\log A_{N_{out}}$, $\log T$, $\log G(T)$, $d(T)$, the combination $d(T) - \log T - \log G(T)$, and the terms $\log \Delta$ and b , respectively. The final subsection combines this information to an indication about the statistics of the r. v. M. Indications about the statistical distribution of most of these terms are based on visual inspection of histograms given for the parameters $\log A_{N_{out}}$, $\log T$, and $\log \Delta$.

1. Statistics of the Maximum Peak-to-Peak Noise Output Amplitude $A_{N_{out}}$

To obtain an indication of the statistical distribution of log amplitude values for the maximum noise amplitude at the seismometer output, the histograms of the log amplitudes actually used in the empirical evaluation part of this study are given in Figures A-1 through A-6. These maximum amplitudes were obtained from a population of noise samples in the passband 17-44 sec. The population was edited to remove extreme values as described in Section III. The maximum amplitude distributions for the VLPE stations CHG, KON, KIP, ALO, and ZLP may be considered to be approximately log-normal. The reason for the somewhat different amplitude behavior at the station CTA is not known at this point, but may be partially caused by insufficient editing.

Another possible indication of the log-normality of maximum noise amplitudes stems from the log-normality of RMS noise levels at the instrument output, observed by Alsup and Becker (1973). In Appendix D it is shown that, if certain assumptions are proven valid, the log-normality of RMS input noise levels implies the log-normality of output maximum noise amplitudes, using the fact that the logarithm of most long period instrument responses is either nearly linear with the period logarithm (VLPE stations), or almost constant (ALPA, NORSAR) in the 20- to 30-sec period range.

Thus, the available data indicate that in general the maximum noise amplitude at the seismometer output can be considered to have an approximately log-normal probability distribution.

2. The Signal Period T

To determine the influence of the period, T , of the maximum signal amplitude, histograms were compiled from VLPE detection data over

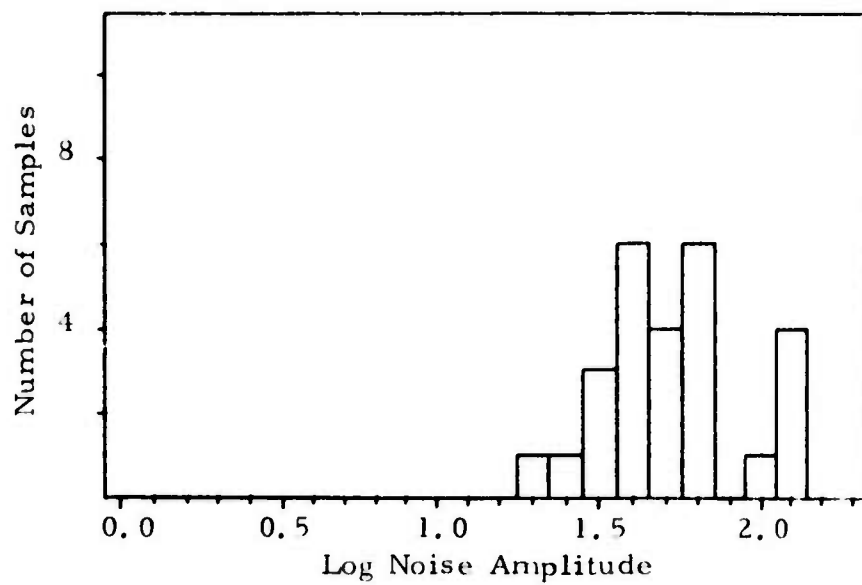


FIGURE A-1

LOG $A_{N_{out}}$ DISTRIBUTION: STATION CTA

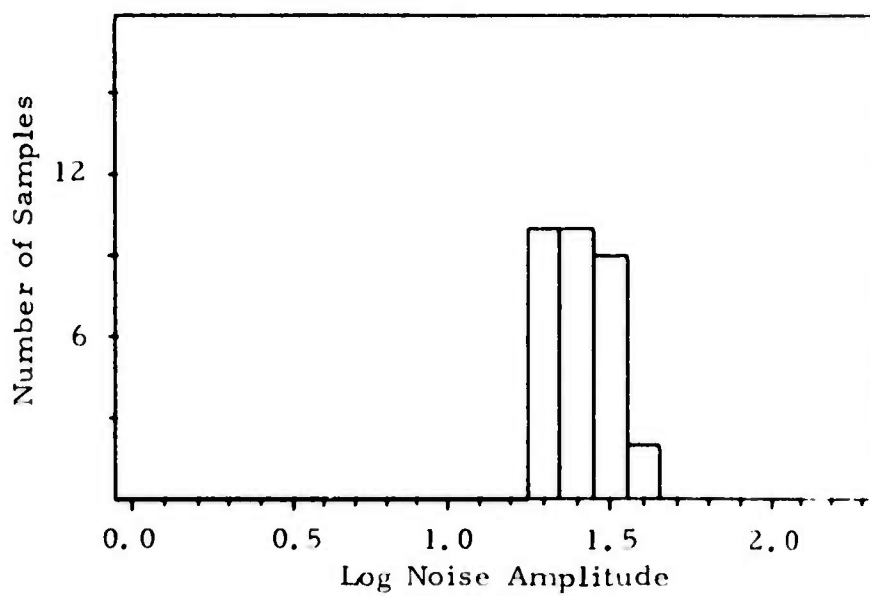


FIGURE A-2

LOG $A_{N_{out}}$ DISTRIBUTION: STATION CHG

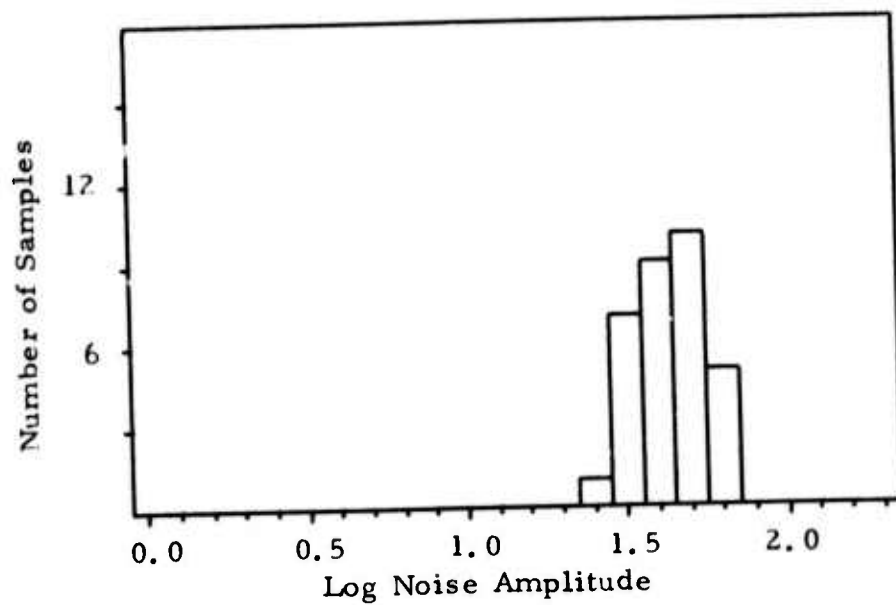


FIGURE A-3

LOG $A_{N_{out}}$ DISTRIBUTION: STATION KON

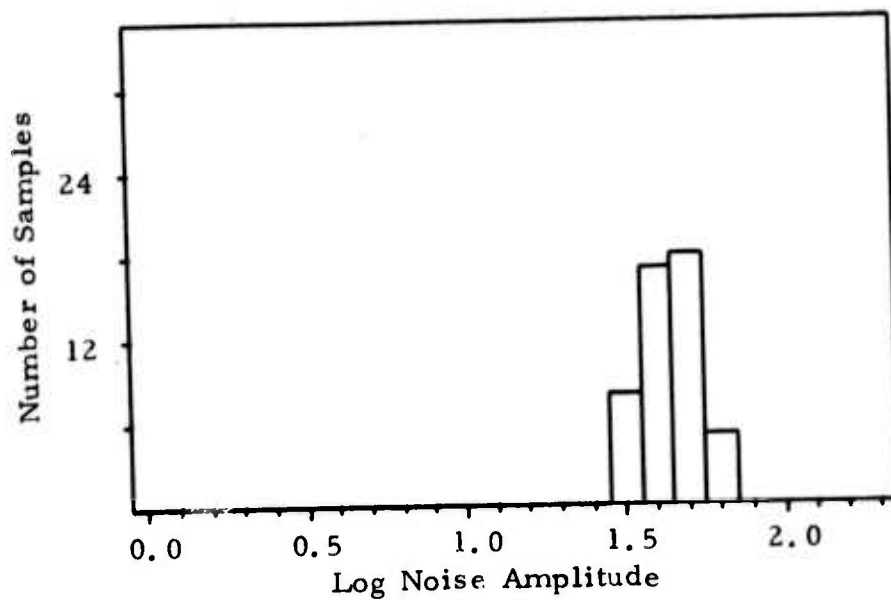


FIGURE A-4

LOG $A_{N_{out}}$ DISTRIBUTION: STATION KIP

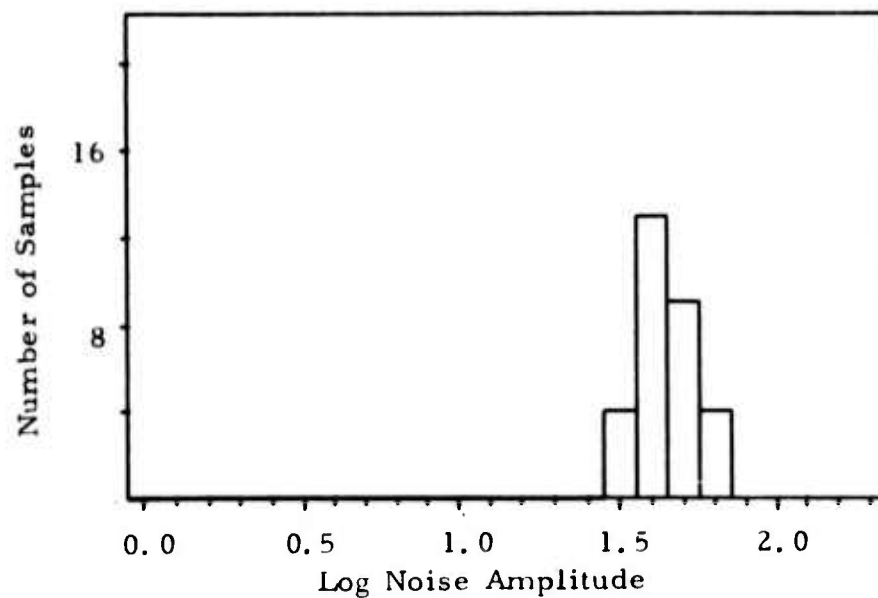


FIGURE A-5

LOG A_{Nout} DISTRIBUTION: STATION ALQ

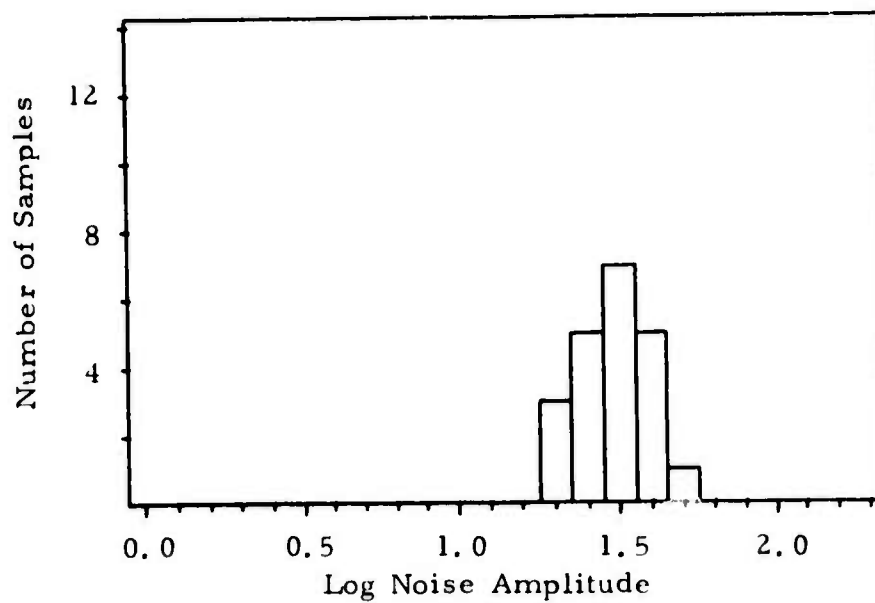


FIGURE A-6

LOG A_{Nout} DISTRIBUTION: STATION ZLP

the year 1972. A typical histogram, the one for station KIP, given in Figure A-7, shows that most maximum signal amplitudes have a period of 30 seconds, and smaller but relatively significant numbers of maximum amplitudes are found at 20, 22, and 28 seconds. This results in the log T histogram of Figure A-8, which indicates that the signal period distribution may be considered to be approximately log-normal. The standard deviation of log T appears to be in the order of 0.1 for most VLPE stations.

3. The Instrument Response $G(T)$

For most VLPE stations the logarithm of the instrument response is approximately linear with the logarithm of the period in the 20- to 30-sec period range (Figure A-9):

$$\log G(T) \approx \alpha \log \frac{T}{20} + \log G(20), \quad 20 \leq T \leq 30, \quad (\text{A-2})$$

where

$$\alpha = \frac{\log G(30) - \log G(20)}{\log 30 - \log 20} = \frac{\log G(30)/G(20)}{0.18}. \quad (\text{A-3})$$

Since a linear transformation of a normal r. v. is again a normal r. v. (e. g., Papoulis, 1965), and log T is approximately normal, the r. v. log G(T) is also approximately normal.

The long period arrays of ALPA and NORSAR have an almost flat response between 20 and 30 seconds, so that there log G(T) is approximately constant and does not affect the statistics of the r. v. M.

4. Statistics of the Magnitude Difference Due to Period Effects, $d(T)$

The difference due to period and path effects combined is plotted in Figure A-10, after data provided by Marshall and Basham (1972). The period is plotted on a logarithmic scale. The curves appear to be approximately linear in segments between 20 and 30 seconds and 30 and 40 seconds.

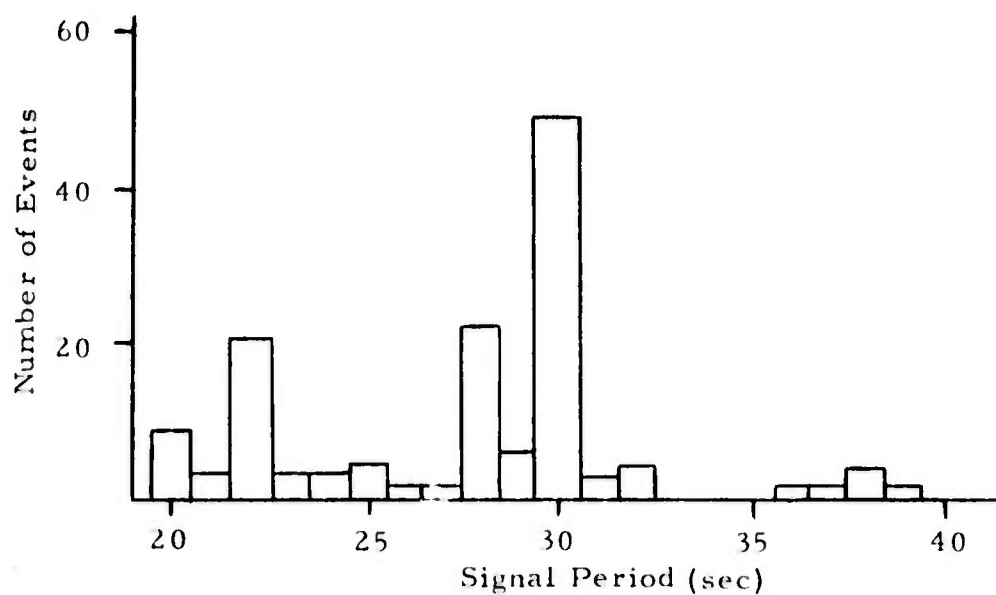


FIGURE A-7
DISTRIBUTION OF SIGNAL PERIODS: STATION KIP

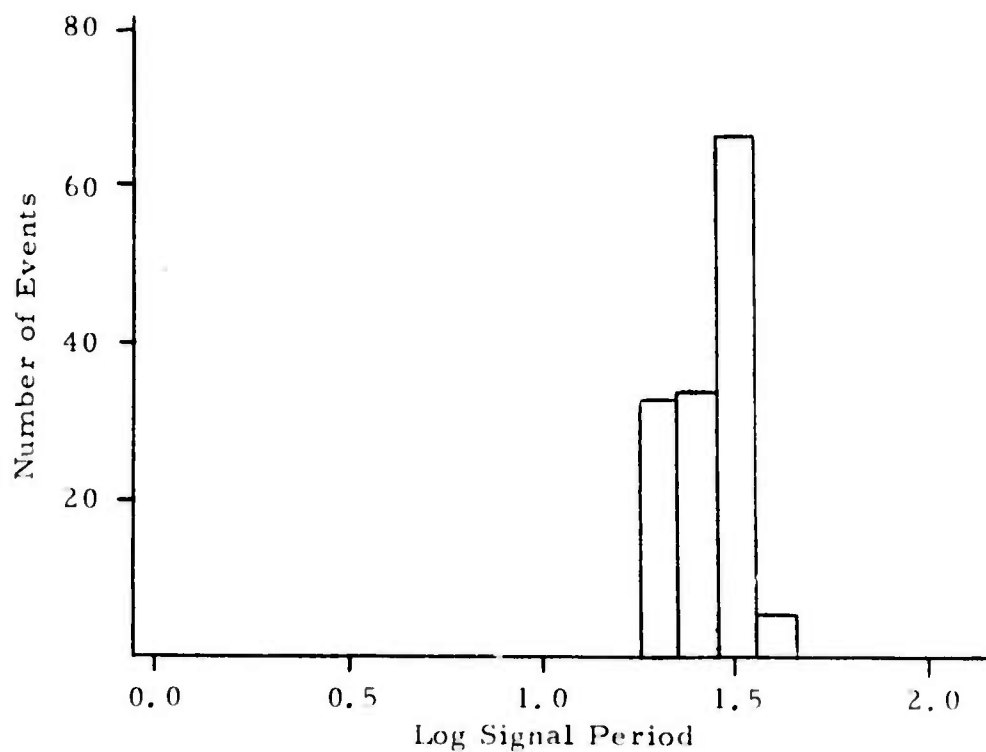
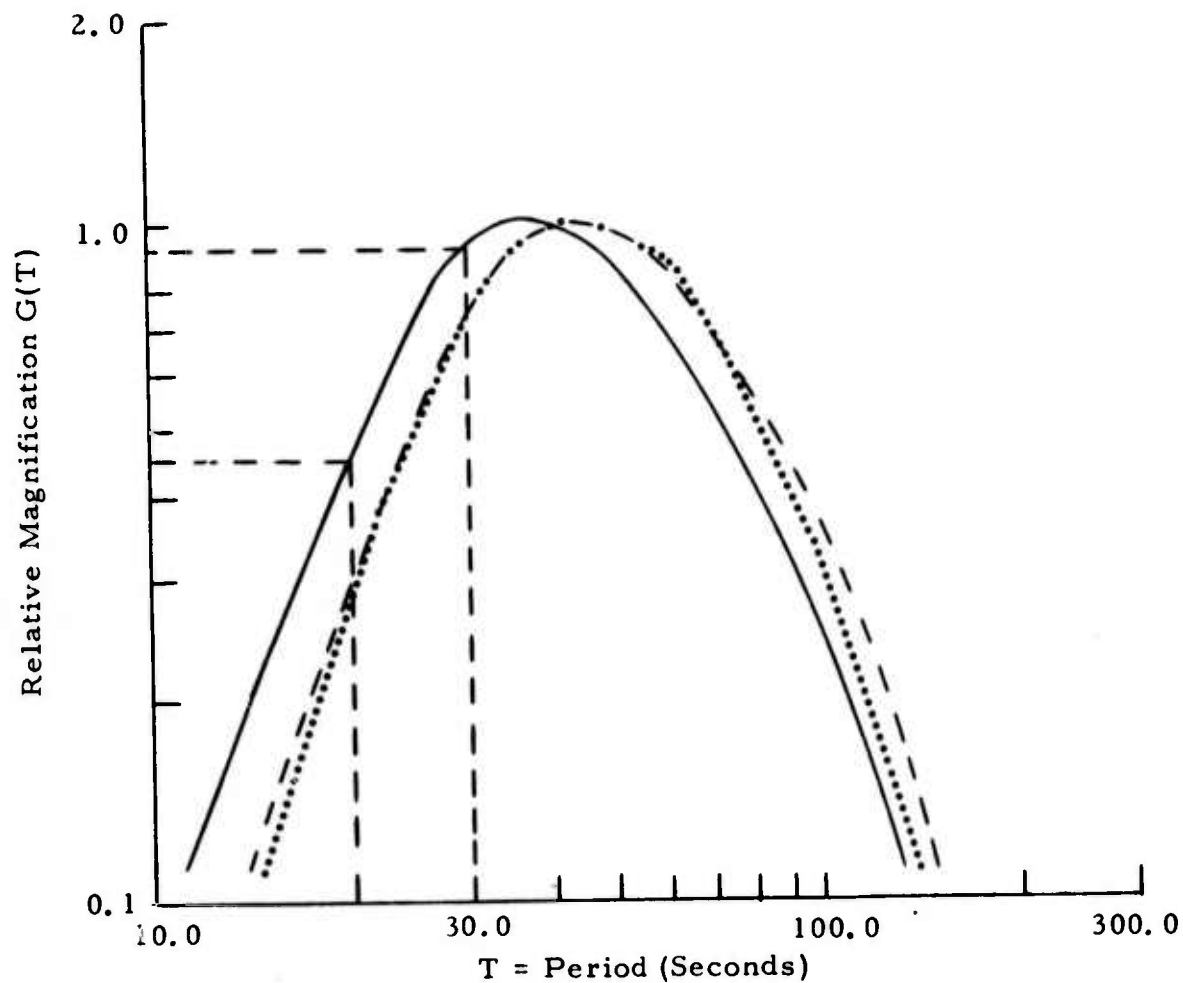


FIGURE A-8
DISTRIBUTION OF SIGNAL PERIOD LOGARITHMS:
STATION KIP



Gain at $T = 40.0 \text{ Sec}$

— Z 0.656 $\text{m}\mu/\text{count}$
 - - - N 0.530 $\text{m}\mu/\text{count}$
 E 0.470 $\text{m}\mu/\text{count}$

FIGURE A-9
 INSTRUMENT RESPONSE FOR KON
 (FROM LAMBERT et al., 1973)

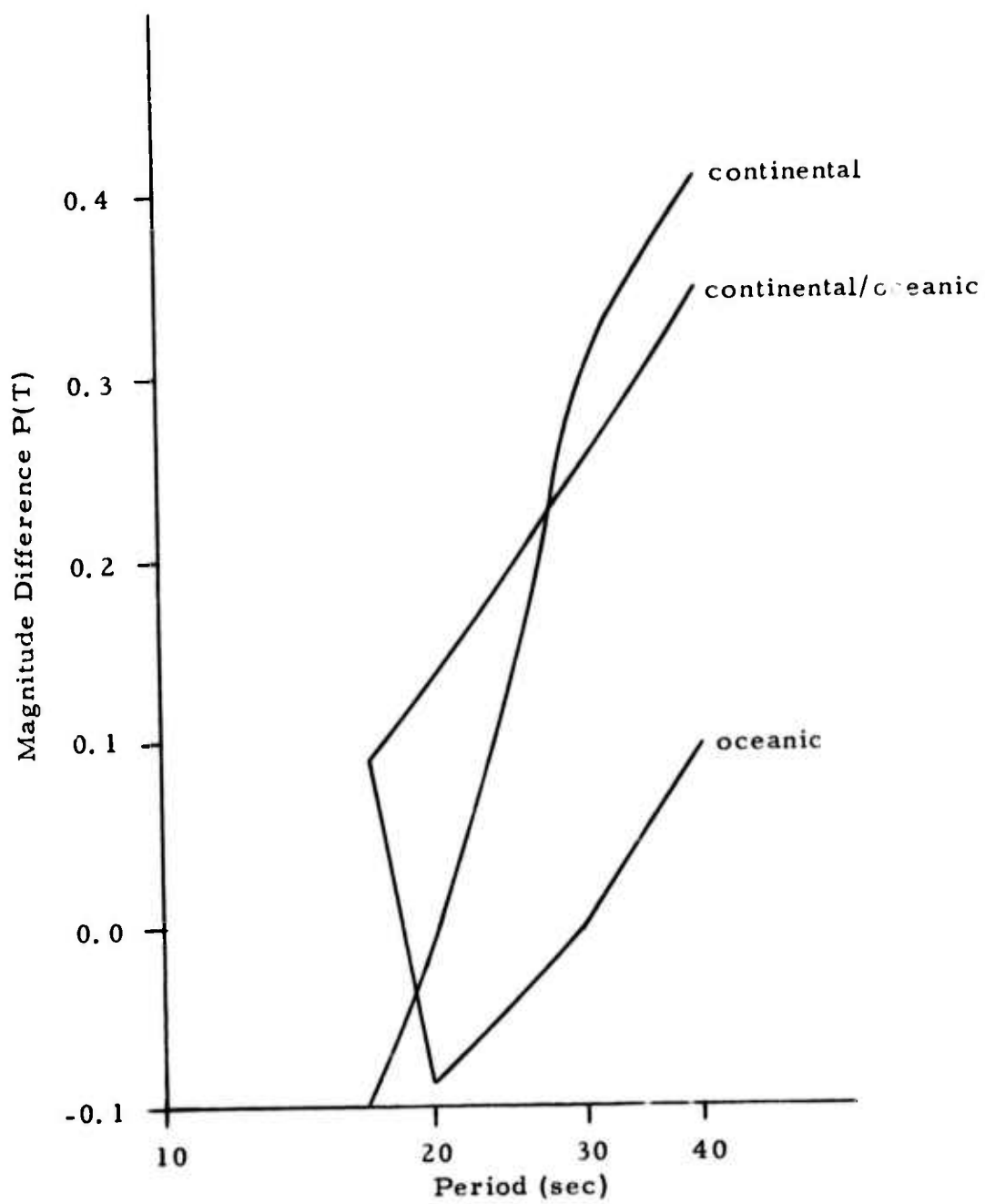


FIGURE A-10
SURFACE WAVE MAGNITUDE DIFFERENCES
DUE TO PATH AND PERIOD EFFECTS
(MARSHALL AND BASHAM, 1972)

Since according to Figure A-7, most signal periods fall in the 20- to 30-sec range, and since the signal period has an approximately log-normal distribution, the magnitude difference will also be approximately normal. The near-linearity with $\log T$ of the magnitude difference $d(T)$ due to period effects only, may be expressed by

$$d(T) \approx \beta \log \frac{T}{20}, \quad 20 \leq T \leq 30. \quad (A-4)$$

For Eurasian continental paths $\beta = 1.70$, for mixed oceanic-continental paths $\beta = 0.68$ according to the Marshall and Basham data.

The path effects shown in these curves presumably are absorbed in the station magnitude bias \bar{b} .

5. Statistics of the Combined Signal Period Effects on M

It now is of interest to study the total effects on the r. v.

$M = M_N + d(T) - b$ due to the statistics of the signal period T . In particular, we are interested in the difference between the T -sec and 20-sec M values determined by the period-dependent terms $\log T$ and $\log G(T)$ in M_N (Equation II-10), and by the term $d(T)$ in Equation (II-15):

$$M(T) - M(20) \approx d(T) - \log \frac{T}{20} - \log \frac{G(T)}{G(20)}, \quad 20 \leq T \leq 30. \quad (A-5)$$

Thus, the empirically determined magnitude difference due to period effects, $d(T)$, is counteracted by the terms $\log T$ and $\log G(T)$.

With Equations (A-3) and (A-4) the above relationship becomes:

$$M(T) - M(20) \approx (\beta - \alpha - 1) \log \frac{T}{20}, \quad 20 \leq T \leq 30. \quad (A-6)$$

Since the combined period effects are approximately linear with $\log T$ the standard deviation of the combined period effects can be expressed in terms of the standard deviation of $\log T$:

$$\sigma_M^T \approx |\beta - \alpha - 1| \sigma_{\log T} \quad , \quad (A-7)$$

where

σ_M^T is the standard deviation of the combined signal period dependent terms in M ;

$\sigma_{\log T}$ is the standard deviation of $\log T$.

Thus the value of σ_M^T depends on the signal period distribution, on the slope of the instrument response between 20 and 30 seconds, and on the rate of magnitude change with the signal period logarithm. For a flat instrument response (ALPA, NORSAR) α equals zero in the above equations.

Table A-1 lists the 20- and 30-sec instrument responses for the period November 1972 through January 1973, and the values α , β , $\beta - \alpha - 1$, and σ_M^T for the VLPE stations used in the empirical evaluation of this study using a $\sigma_{\log T}$ value of 0.1 (Figure A-8). The stations CHG, KON, ALQ, and ZLP thereby are assumed to receive most Eurasian event signals over continental propagation paths; the paths to the stations CTA and KIP are in general mixed continental-oceanic. We notice that the $\log TG(T)$ term always overcompensates the term $d(T)$; the extent of overcompensation depends on the factors mentioned above.

6. The Epicentral Distance Δ

We now turn to the next variable in the noise magnitude, the epicentral distance Δ . For a relatively large area we would not expect the parameter Δ to have a log-normal distribution. For example, for the population of events (detected and undetected) processed by Lambert et al. (1973) the $\log \Delta$ distribution of the VLPE stations is given in Figures A-11 through A-22. For instance, for KIP (Figure A-18), the distances range from 14° to 146° , and their logarithms accordingly from 1.4 to 2.2. In processing the

TABLE A-1
SIGNAL PERIOD DEPENDENT PARAMETERS FOR VLPE STATIONS

Station	G(20)	G(30)	α	β	$\beta - \alpha - 1$	σ_M^T
CTA	0.245	0.810	2.95	0.68	-3.27	0.33
CHG	0.225	0.660	2.55	1.70	-1.85	0.18
KON	0.460	0.920	1.70	1.70	-1.00	0.10
KIP	0.510	0.990	1.64	0.68	-1.96	0.20
ALQ	0.485	1.000	1.79	1.70	-1.09	0.11
ZLP	0.365	0.890	2.20	1.70	-1.50	0.15

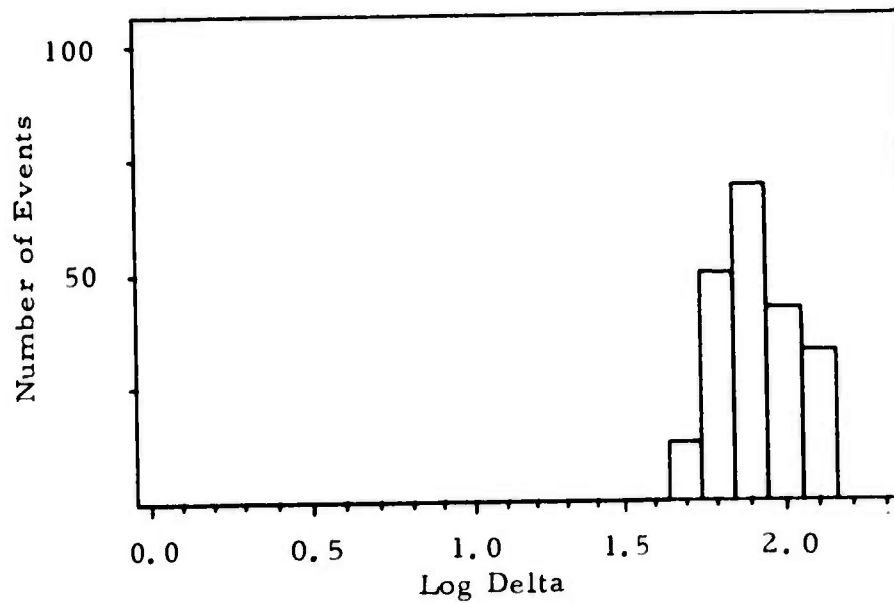


FIGURE A-11
LOG Δ DISTRIBUTION: STATION CTA

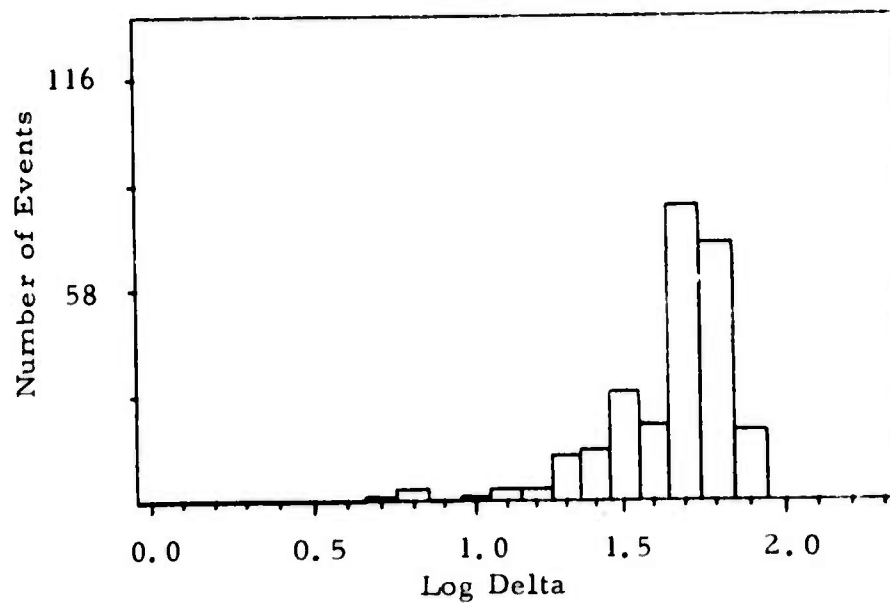


FIGURE A-12
LOG Δ DISTRIBUTION: STATION CHG

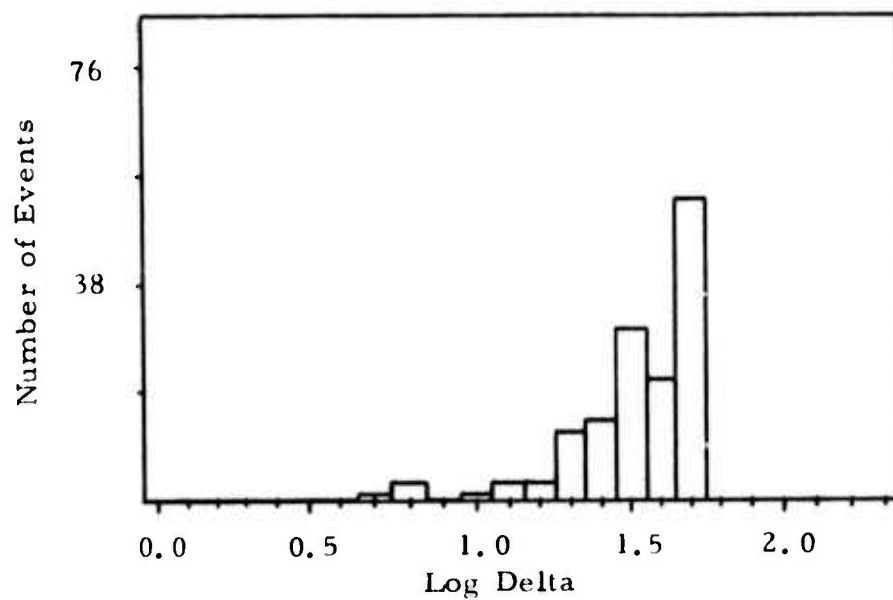


FIGURE A-13
LOG Δ DISTRIBUTION, $\Delta < 50^\circ$: STATION CHG

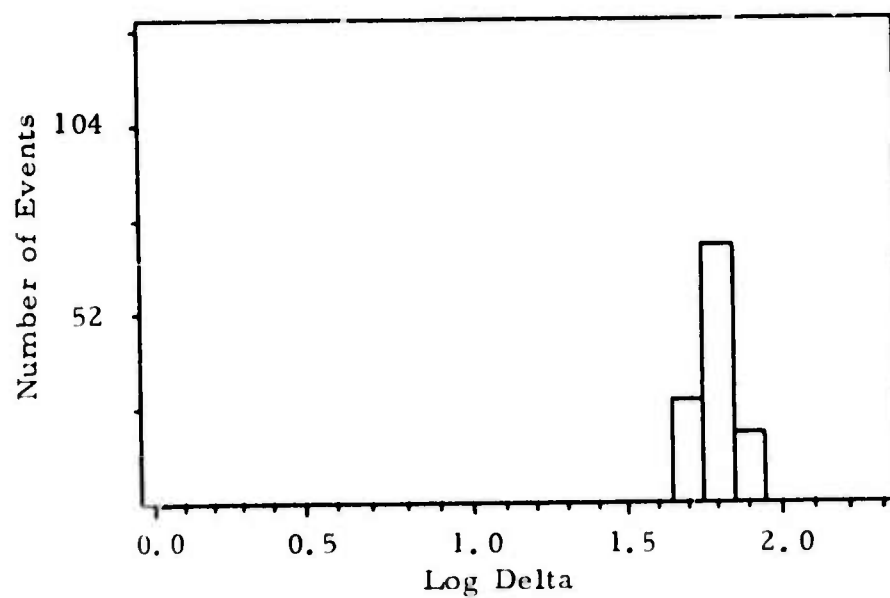


FIGURE A-14
LOG Δ DISTRIBUTION, $\Delta \geq 50^\circ$: STATION CHG

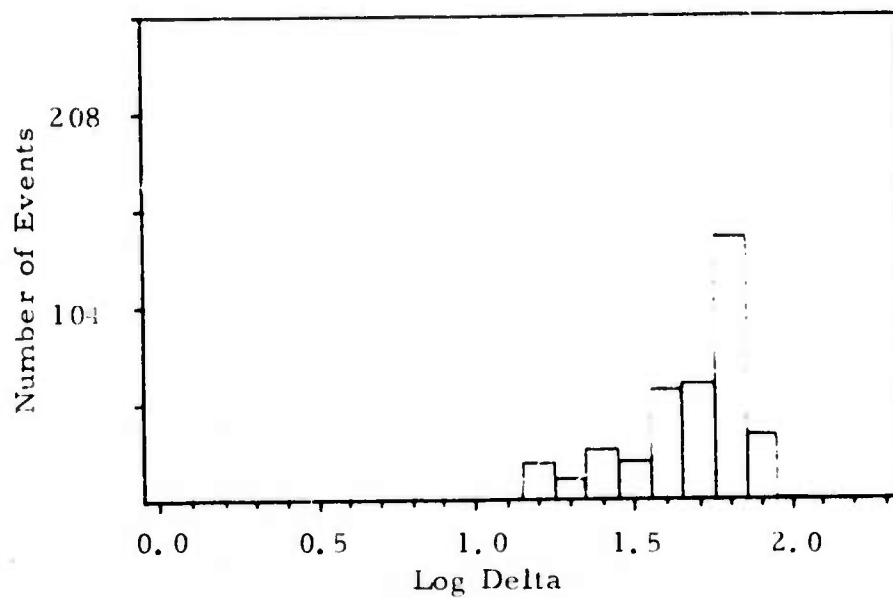


FIGURE A-15

LOG Δ DISTRIBUTION: STATION KON

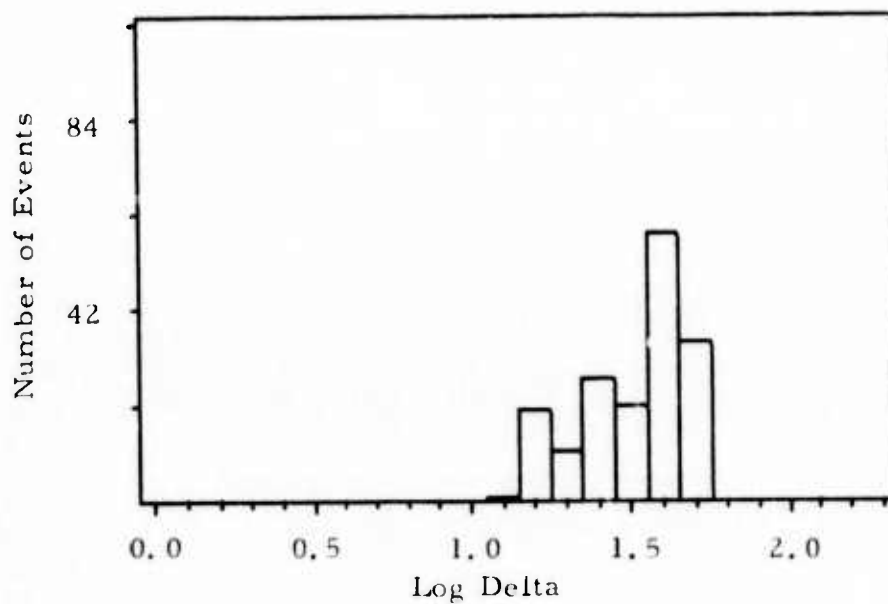


FIGURE A-16

LOG Δ DISTRIBUTION, Δ < 50°: STATION KON

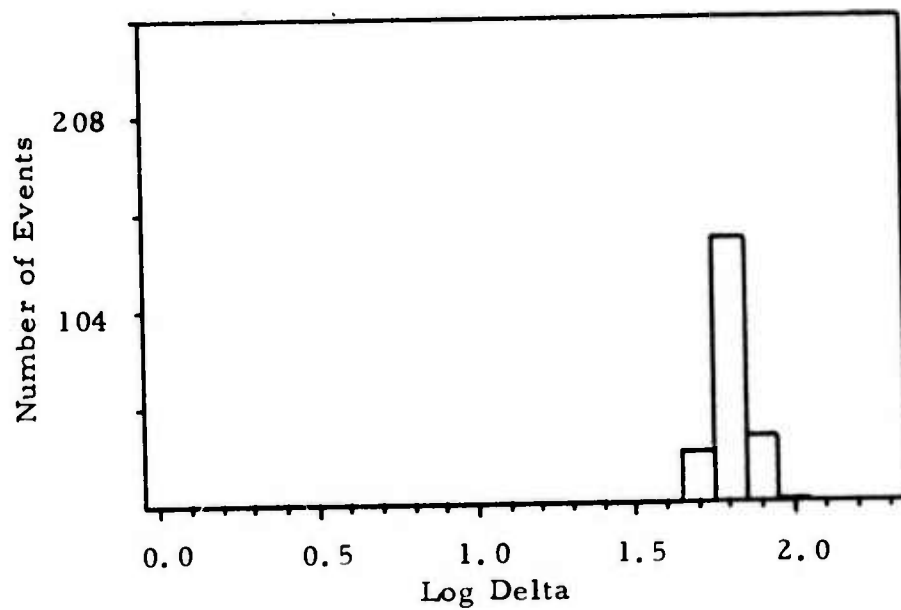


FIGURE A-17
LOG Δ DISTRIBUTION, $\Delta \geq 50^\circ$: STATION KON

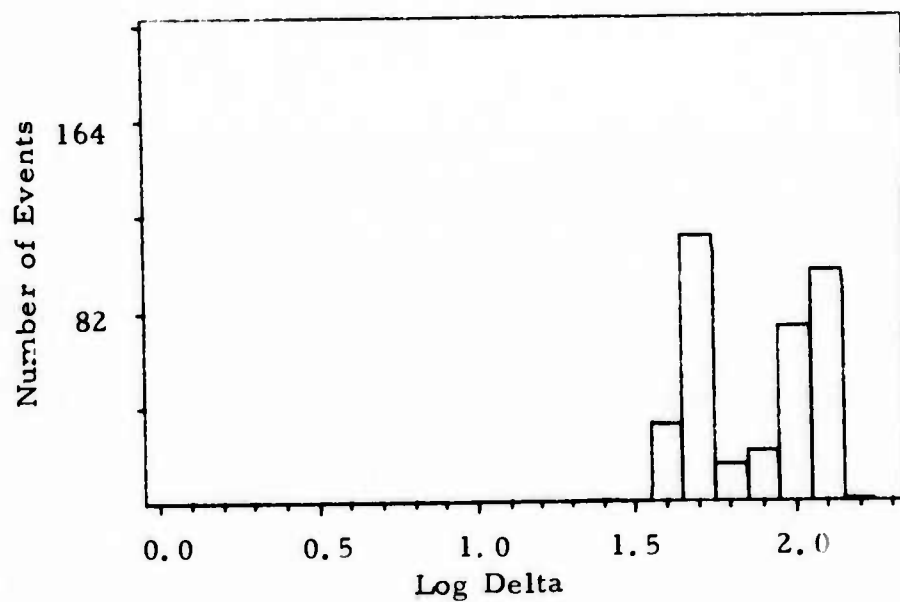


FIGURE A-18
LOG Δ DISTRIBUTION: STATION KIP

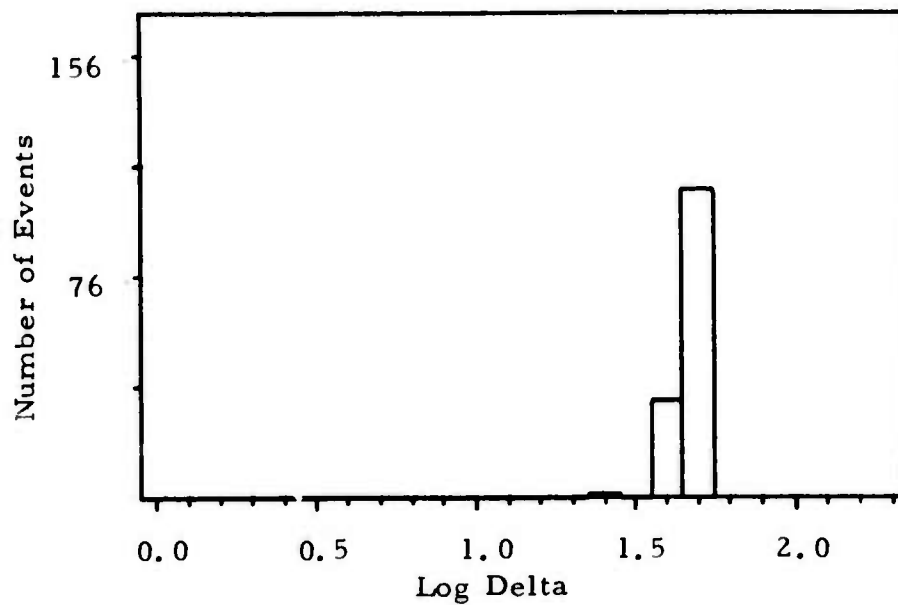


FIGURE A-19
LOG Δ DISTRIBUTION, $\Delta < 50^\circ$: STATION KIP

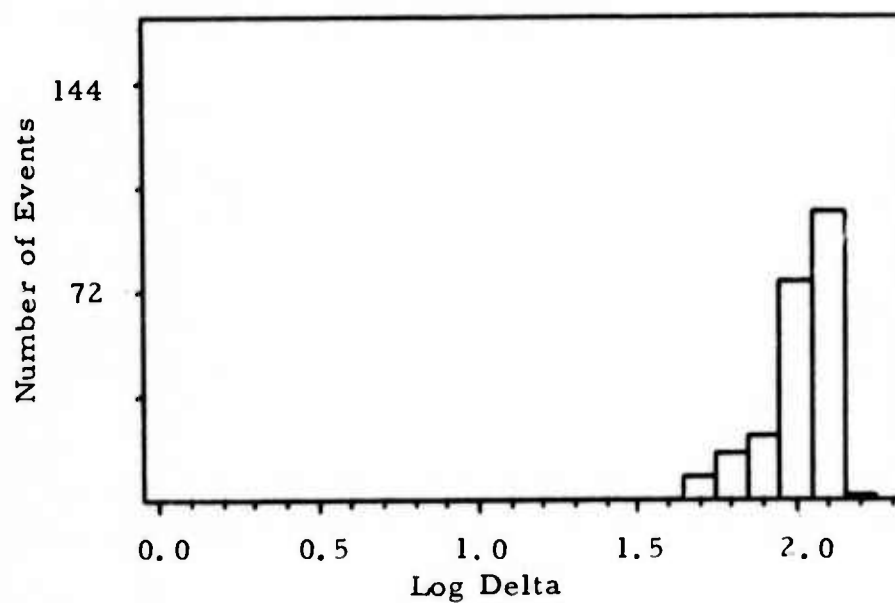


FIGURE A-20
LOG Δ DISTRIBUTION, $\Delta \geq 50^\circ$: STATION KIP

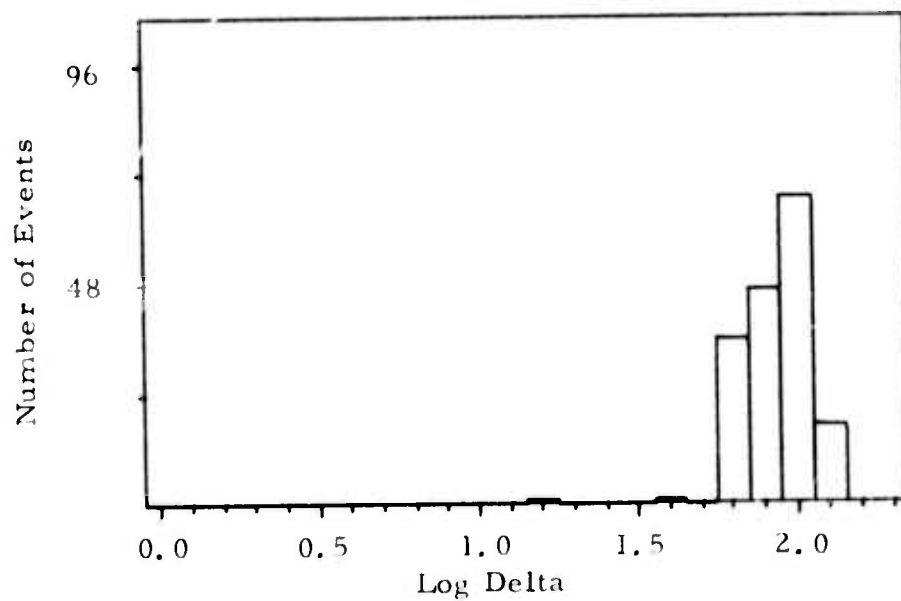


FIGURE A-21
LOG Δ DISTRIBUTION: STATION ALQ

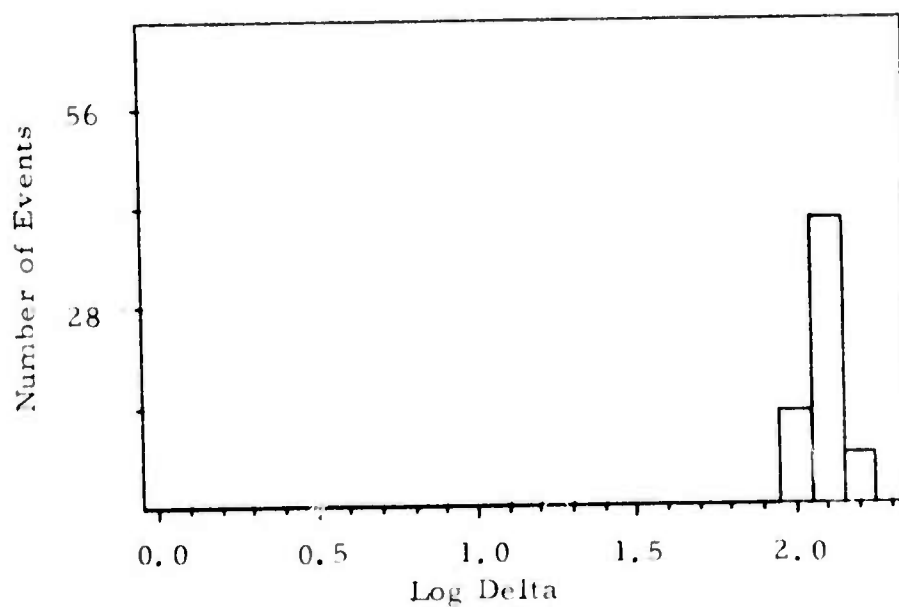


FIGURE A-22
LOG Δ DISTRIBUTION: STATION ZLP

events Lambert et al. (1973) split the Δ -populations into groups of events with epicentral distances either greater or less than 50° . For KIP these distributions are given in Figure A-19 and Figure A-20, respectively. The bin size causes an overlap between the two groups at $\log 50^\circ = 1.7$.

It is noticed that decreasing the Δ -range in the above manner leads only in a few cases to approximation of a log-normal distribution. Thus, if a log-normal Δ -distribution is desired for a certain detectability estimation model, a relatively narrow distance range must be considered. In the case of direct detectability estimation this may cause the population of events to drop below the statistically required minimum. When estimating the detectability from noise, however, we may compute detectability thresholds for one particular distance value Δ_0 or for a relatively narrow Δ -range centered about some Δ_0 , and shift the thresholds over an amount $\log \Delta/\Delta_0$ for other Δ -values of interest. The variance of M due to distance variation, of course increases with the range of distances, thereby changing the shape of the detectability curve $P(\det \bar{M}_s(20) = x) = F_M(x)$.

7. The Station Magnitude Variation b

The station magnitude variation, b , about the reference magnitude is described by Equation (II-5). The mean variation is called the station magnitude bias. Bodywave magnitudes were found to be approximately normally distributed (Freedman, 1967). For detectability estimation modeling purposes we will assume the variation of the surface-wave magnitudes also to be normal; however, this needs further confirmation.

8. Total Statistics of the Random Variable $M = M_N + d(T) - b$

With the statistics known on each of the terms involved in the r.v. M we are now able to determine the total statistics of this variable. We found that each term, except the epicentral distance, was approximately normally distributed, while the epicentral distance would either be kept fixed, or

allowed to be taken only over a narrow range to approximate a log-normal distribution. The linear combination M of these approximately normally distributed random variables then again establishes approximately a normal r.v..

The next point is to check if the parameters $A_{N_{out}}$, T , Δ , and b are correlated. The maximum noise amplitude at the seismometer output, $A_{N_{out}}$ is not correlated with any of the parameters T , Δ , or b . The signal period T may be slightly correlated with Δ because of path effects. For long-period signals and small event areas, however, this correlation is expected to be low. There may also be some correlation between T and b , since the station magnitude variation is caused by radiation pattern and propagation path effects. Also this correlation is expected to be low, however. Finally, for the same reason, there may be some correlation between Δ and b . A thorough study of these correlations would be desirable but we suggest that this be done as a separate topic.

If all variables are considered to be mutually uncorrelated, the standard deviation of the r.v. M can be computed from

$$\sigma_M^2 = \sigma_{\log A}^2 + (\beta - \alpha - 1)^2 \sigma_{\log T}^2 + \sigma_{\log \Delta}^2 + \sigma_b^2, \quad (A-8)$$

where

- σ_M is the standard deviation of the r.v. M ;
- $\sigma_{\log A}$ is the standard deviation of $\log A_{N_{out}}$;
- $\sigma_{\log T}$ is the standard deviation of $\log T$;
- $\sigma_{\log \Delta}$ is the standard deviation of $\log \Delta$;
- σ_b is the station magnitude bias standard deviation.

The standard deviation of the logarithm of the maximum noise amplitude at the seismometer output, $\sigma_{\log A}$, must be determined empirically

from noise sample populations. For well-performing VLPE stations, this value turns out to be 0.1 to 0.2 (Section III and Appendix E). The standard deviation of the signal period logarithm, $\sigma_{\log T}$, was found to be approximately 0.1. The standard deviation of the combined period effects then follows from Equation (A-7); their values were listed in Table A-1, and range from 0.10 to 0.33 depending on the kind of path, on the slope of the instrument response, and on the slope of the variable $d(T)$.

The standard deviation of the event epicentral distance logarithm depends on the relative distance range, i.e., it depends on the size of the area of interest and on the distance between that area and the station concerned. For most VLPE stations and Eurasian events $\sigma_{\log \Delta}$ ranges from 0.1 to 0.2; the $\log \Delta$ distribution is non-normal for such a large relative distance range. For a fixed Δ -value, naturally, $\sigma_{\log \Delta} = 0$; it is small for a narrow relative distance range. In the empirical evaluation of this study the Δ ranges were chosen the same as in the maximum likelihood direct detectability estimation reported by Lambert et al. (1973). The $\log \Delta$ distributions for these cases were given in Figures A-11 through A-22. The station magnitude standard deviation, finally, according to this same reference, was calculated at approximately 0.35.

Assuming no correlation among the parameters, $A_{N_{out}}$, T , Δ , and b , the standard deviation of the r.v. M then is in the order of

$$\sigma_M \approx \left[(1.0 \text{ to } 0.2)^2 + (0.10 \text{ to } 0.33)^2 + (0.1 \text{ to } 0.2)^2 + 0.35^2 \right]^{1/2} \quad (\text{A-9})$$

or

$$0.39 \leq \sigma_M \leq 0.55. \quad (\text{A-10})$$

The above is only a preliminary determination of the statistics involved. More detailed studies should be performed, both analytically and empirically, to further test the assumptions, observations, and calculations.

APPENDIX B
VLPE LOG NOISE PERIOD DISTRIBUTION

The following pages contain the distributions of the logarithms of the maximum noise amplitude periods.

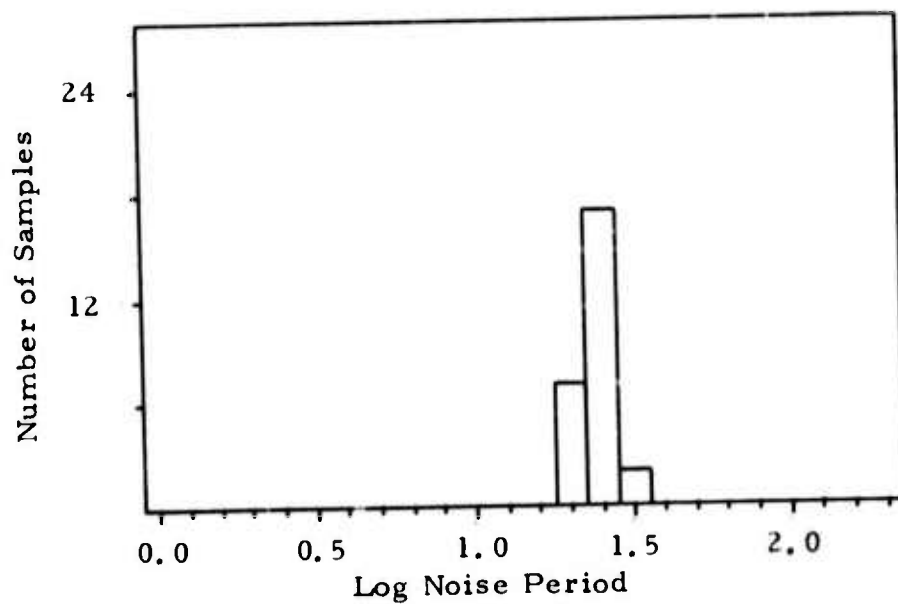


FIGURE B-1

LOG NOISE PERIOD DISTRIBUTION: STATION CTA

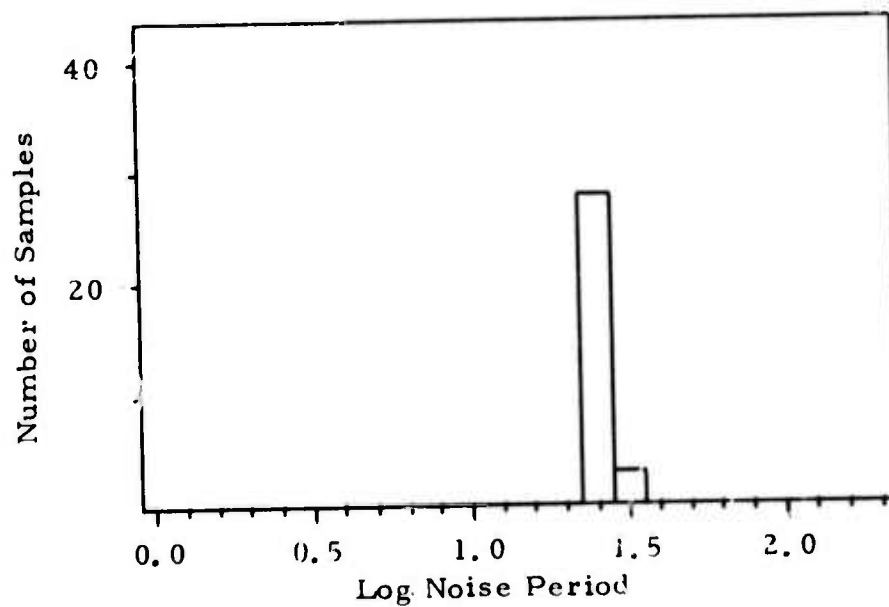


FIGURE B-2

LOG NOISE PERIOD DISTRIBUTION: STATION CHG

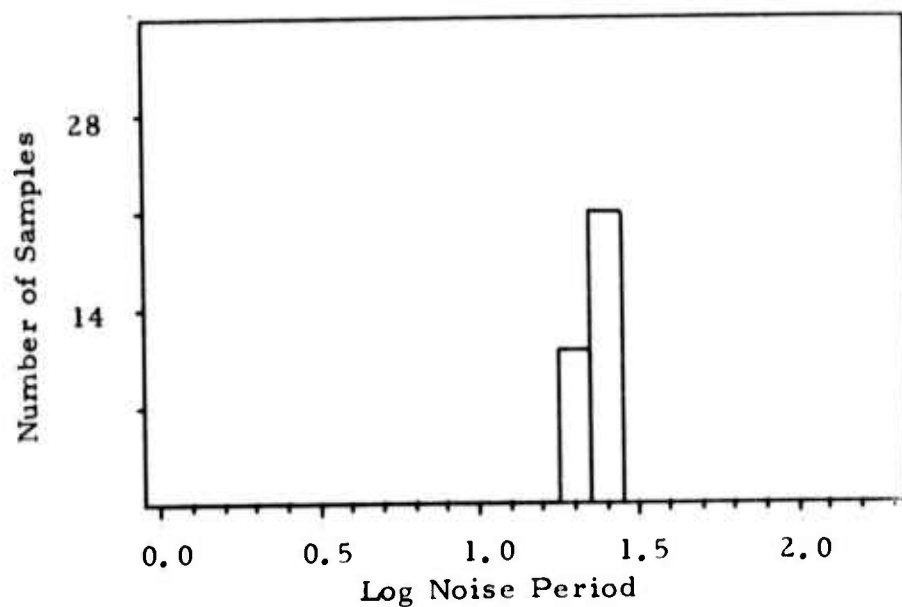


FIGURE B-3

LOG NOISE PERIOD DISTRIBUTION: STATION KON

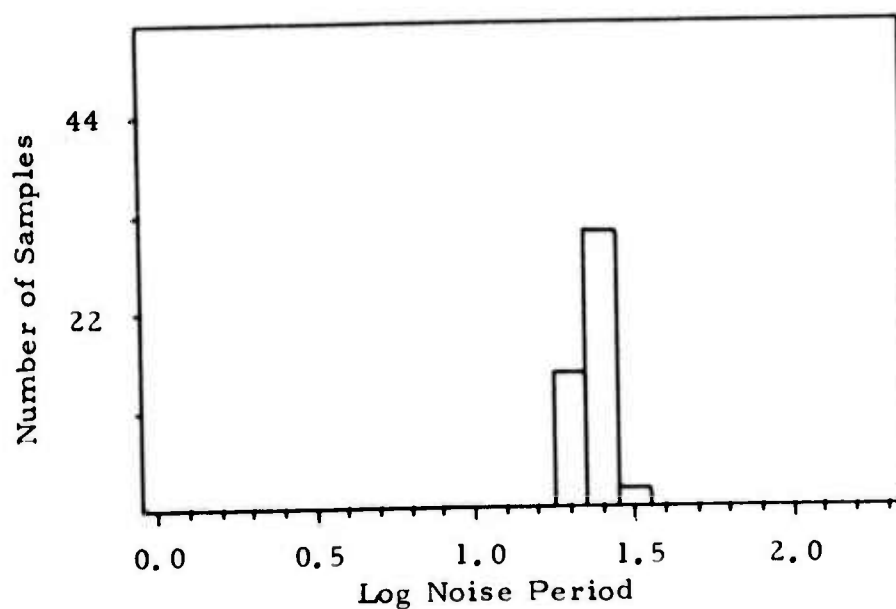


FIGURE B-4

LOG NOISE PERIOD DISTRIBUTION: STATION KIP

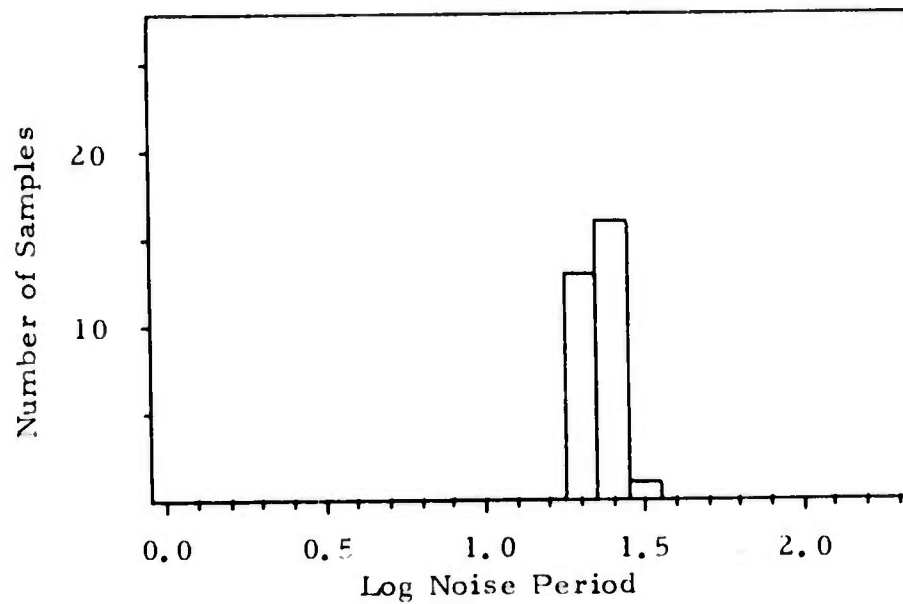


FIGURE B-5

LOG NOISE PERIOD DISTRIBUTION: STATION ALQ

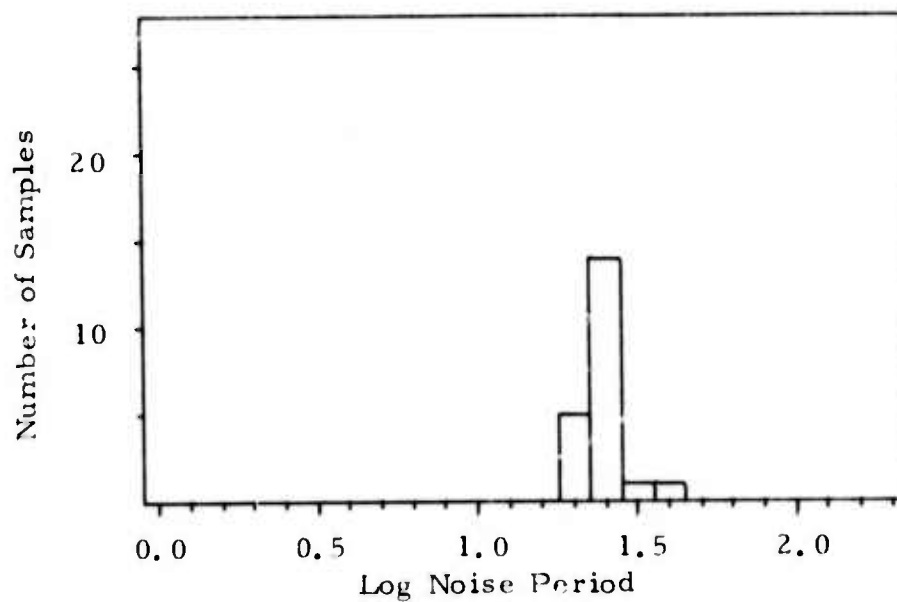


FIGURE B-6

LOG NOISE PERIOD DISTRIBUTION: STATION ZLP

APPENDIX C

SOME CHARACTERISTICS OF THE LOGARITHM

Below we will develop some characteristics of the logarithm which are of interest in the computation of magnitudes of seismic events and their averages.

1. Variation of $\log x$ Due To Variation Of x

Consider the variable:

$$x = x_0 + \Delta x = x_0 \left(1 + \frac{\Delta x}{x_0} \right) , \quad (C-1)$$

then

$$\log x = \log x_0 + \log \left(1 + \frac{\Delta x}{x_0} \right) . \quad (C-2)$$

Thus, the variation in $\log x$, due to the variation in x , is determined by the relative change, $\Delta x/x_0$, and equals

$$\Delta(\log x) = \log \left(1 + \frac{\Delta x}{x_0} \right) . \quad (C-3)$$

For $\left| \frac{\Delta x}{x_0} \right| < 1$ the right-hand side may be developed into a Taylor series, so that Equation (C-2) becomes

$$\begin{aligned} \log x = \log x_0 + 0.43 \left[\frac{\Delta x}{x_0} - \frac{1}{2!} \left(\frac{\Delta x}{x_0} \right)^2 \right. \\ \left. + \frac{1}{3!} \left(\frac{\Delta x}{x_0} \right)^3 - \dots \right] . \end{aligned} \quad (C-4)$$

For small values of $\frac{\Delta x}{x_0}$ the variation of $\log x$ with x is approximately linear:

$$\log x \approx \log x_0 + 0.43 \frac{\Delta x}{x_0} . \quad (C-5)$$

For a value $\frac{\Delta x}{x_0} < 0.5$, for instance, the error is less than 0.05. The linear approximation and the exact function of the term $\log(1 + \frac{\Delta x}{x_0})$ are plotted in Figure C-1.

2. Mean-of-the-Logarithm Versus Logarithm-of-the-Mean

Assume a r. v. x to vary symmetrically about its mean, μ , with a standard deviation σ :

$$x = \mu + \Delta x = \mu(1 + \frac{\Delta x}{\mu}) . \quad (C-6)$$

Applying Equation (C-4) results in

$$\begin{aligned} \log x = \log \mu + 0.43 \left[\frac{\Delta x}{\mu} - \frac{1}{2!} \left(\frac{\Delta x}{\mu} \right)^2 \right. \\ \left. + \frac{1}{3!} \left(\frac{\Delta x}{\mu} \right)^3 - \dots \right] . \end{aligned} \quad (C-7)$$

To obtain the mean of the logarithm we take the expected value on both sides:

$$\begin{aligned} \mu_{\log x} = \log \mu + 0.43 E \left[\frac{\Delta x}{\mu} - \frac{1}{2!} \left(\frac{\Delta x}{\mu} \right)^2 \right. \\ \left. + \frac{1}{3!} \left(\frac{\Delta x}{\mu} \right)^3 - \dots \right] . \end{aligned} \quad (C-8)$$

Since for a symmetric distribution of Δx about μ $E(\Delta x) = 0$ and $E(\Delta x)^2 = \sigma^2$ this becomes

$$\mu_{\log x} = \log \mu - 0.215 \left(\frac{\sigma}{\mu} \right)^2 + \text{higher order terms.} \quad (C-9)$$

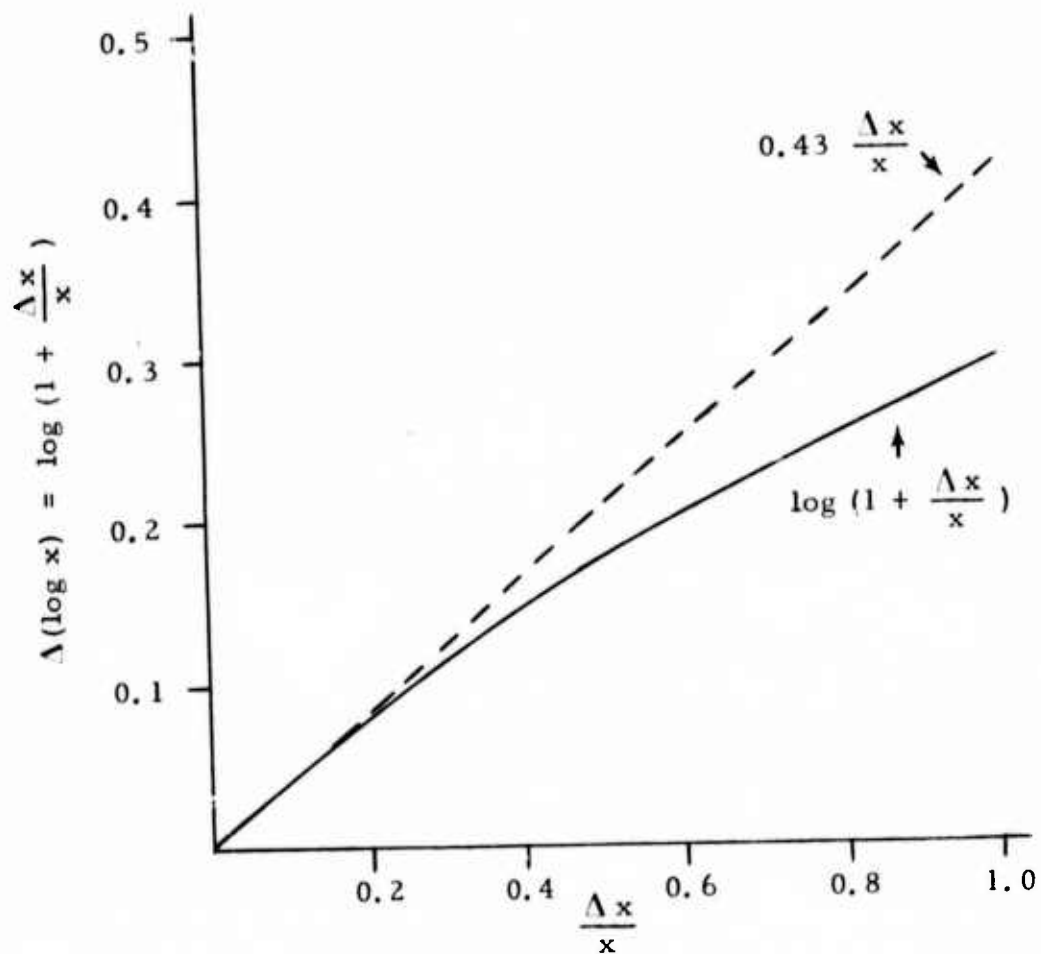


FIGURE C-1
VARIATION IN $\log x$, DUE TO VARIATION IN x

Thus, for small σ/μ values the arithmetic mean of the logarithm approximately equals the logarithm of the arithmetic mean; the error term is approximately

$$\epsilon \approx 0.215 \left(\frac{\sigma}{\mu} \right)^2 . \quad (C-10)$$

For instance, for a ratio of $\sigma/\mu = 0.5$ the error term is approximately 0.05.

The validity of the above approximation, and in particular the accuracy of the error term, depend on the rate of convergence of the higher order terms, which is a matter of third, fourth, etc. statistical moments. According to the results presented in Table III-2 the approximation of the error term evidently is very good.

APPENDIX D

DISTRIBUTION OF INPUT VERSUS OUTPUT NOISE AMPLITUDES

Alsup and Becker (1973) found that the RMS true ground motion noise levels were approximately log-normally distributed. Lambert et al. (1973), observed that at the seismometer output the maximum-noise-amplitude-over-RMS ratio was relatively constant for VLPE data. We will show that if this ratio is also constant at the instrument input, then the output RMS noise levels and maximum noise amplitudes are also approximately log-normally distributed.

The input and output amplitudes and RMS values are related by:

$$\begin{aligned} \log A_{N_{out}} = & \log RMS_{in} + \log (A_{N_{in}}/RMS_{in}) \\ & + \log G(T_N) , \end{aligned} \quad (D-1)$$

where

$A_{N_{out}}$ is the maximum peak-to-peak noise output amplitude;

$A_{N_{in}}$ is the maximum peak-to-peak noise input amplitude;

RMS_{in} is the RMS input noise;

$G(T_N)$ is the instrument response for the period T_N of the maximum noise amplitude.

According to the tables in Appendix E, and the histograms in Appendix B, the noise periods T_N fall mainly within the 20- to 30-sec range and can be considered to be approximately log-normally distributed. Since

for the 20- to 30-sec range $\log G(T_N)$ is approximately linear with $\log T_N$, also $G(T_N)$ will be approximately log-normally distributed. If the input amplitude-over-RMS-noise ratio is constant, the right-hand side of Equation (D-1) is a linear combination of two normally distributed random variables plus a constant, so that $\log A_{N_{out}}$ must be approximately normally distributed. Since, furthermore,

$$\log A_{N_{out}} = \log RMS_{out} + \log (A_{N_{out}}/RMS_{out}), \quad (D-2)$$

and the $A_{N_{out}}/RMS_{out}$ ratio was found to be relatively constant, the output RMS noise values also are log-normally distributed.

APPENDIX E
NOISE DATA PROCESSING TABLES

The tables on the following pages display part of the noise data processing as described in Section III.

TABLE E-1

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION CTA

STATION 1 CTA 17 - 44 SEC, PDIT NO. 4, SEISMOMETER OUTPUT								
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO
1	72	307	121.67	10.55	11.53	2.09	1.02	1.06
2	72	321	64.67	6.79	9.54	1.81	0.83	0.98
3	72	323	28.62	4.10	6.98	1.46	0.61	0.84
4	72	325	45.93	5.64	8.14	1.66	0.75	0.91
5	72	327	38.81	5.51	7.04	1.59	0.74	0.85
6	72	329	37.52	5.67	6.62	1.57	0.75	0.82
7	72	330	62.36	10.79	5.78	1.79	1.03	0.76
8	72	331	30.88	4.27	7.23	1.49	0.63	0.86
9	72	334	116.06	9.12	12.73	2.06	0.96	1.10
10	72	335	119.44	11.06	10.80	2.08	1.04	1.03
11	72	346	54.90	4.74	11.58	1.74	0.68	1.06
12	72	347	90.37	10.67	8.47	1.96	1.03	0.93
13	72	348	54.44	8.33	6.54	1.74	0.92	0.82
14	72	349	66.30	10.22	6.49	1.82	1.01	0.81
15	72	350	21.35	3.50	6.10	1.33	0.54	0.79
16	72	351	38.68	4.89	7.91	1.59	0.69	0.90
17	72	352	53.72	5.81	9.25	1.73	0.76	0.97
18	72	353	39.02	4.81	8.11	1.59	0.68	0.91
19	72	354	28.44	3.92	7.26	1.45	0.59	0.86
20	72	355	62.18	5.05	12.31	1.79	0.70	1.09
21	72	356	56.83	5.50	10.33	1.75	0.74	1.01
22	72	357	43.65	6.47	6.75	1.64	0.81	0.83
23	72	359	56.29	6.58	8.55	1.75	0.82	0.93
24	72	360	120.32	9.74	12.35	2.08	0.99	1.09
25	72	362	27.04	3.15	8.58	1.43	0.50	0.93
26	72	364	40.71	5.98	6.81	1.61	0.78	0.93
MEAN			58.47	6.65	8.61	1.72	0.79	0.92
LOG MEAN			1.77	0.82	0.93			
STD DEV			30.51	2.53	2.14	0.21	0.16	0.10
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN +OR- 2*STD.DEV.)								

TABLE E-2

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION CHG

STATION 2 CHG 17 - 44 SEC, EDIT NO. 11, SEISMOMETER OUTPUT									
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO	AMP PER
1	72	308	28.00	3.53	7.93	1.45	0.55	0.90	32
2	72	309	22.28	3.45	6.46	1.35	0.54	0.81	28
3	72	311	20.88	3.67	5.69	1.32	0.56	0.76	24
4	72	318	34.12	4.51	7.57	1.53	0.65	0.88	24
5	72	325	19.36	2.81	6.89	1.29	0.45	0.84	24
6	72	326	24.44	3.73	6.55	1.39	0.57	0.82	28
7	72	327	29.93	4.00	7.48	1.48	0.60	0.87	28
8	72	328	18.95	3.12	6.07	1.28	0.49	0.78	24
9	72	333	34.05	3.92	8.69	1.53	0.59	0.94	24
10	72	334	30.44	3.52	8.65	1.48	0.55	0.93	24
11	72	342	31.69	4.41	7.19	1.50	0.64	0.86	24
12	72	344	35.29	4.01	8.80	1.55	0.60	0.94	24
13	72	345	22.36	3.69	6.06	1.35	0.57	0.78	32
14	73	2	26.84	4.32	6.21	1.43	0.64	0.79	28
15	73	6	23.15	3.45	6.71	1.36	0.54	0.83	24
16	73	7	29.58	3.90	7.58	1.47	0.59	0.88	24
17	73	8	38.55	4.01	9.61	1.59	0.60	0.98	24
18	73	11	23.05	2.83	8.14	1.36	0.45	0.91	28
19	73	13	25.07	3.88	6.46	1.40	0.59	0.81	28
20	73	14	39.16	4.09	9.57	1.59	0.61	0.98	24
21	73	15	26.27	4.23	6.21	1.42	0.63	0.79	28
22	73	17	18.02	3.03	5.95	1.26	0.48	0.77	28
23	73	19	29.89	3.09	9.67	1.48	0.49	0.99	24
24	73	20	22.29	2.89	7.71	1.35	0.46	0.89	28
25	73	21	26.45	3.17	8.34	1.42	0.50	0.92	24
26	73	24	27.49	3.74	7.35	1.44	0.57	0.87	28
27	73	26	19.22	3.23	5.95	1.28	0.51	0.77	28
28	73	27	21.92	4.01	5.47	1.34	0.60	0.74	32
29	73	28	20.29	3.00	6.76	1.31	0.48	0.83	24
30	73	29	29.79	3.60	8.27	1.47	0.56	0.92	24
31	73	30	25.56	3.39	7.54	1.41	0.53	0.88	24
MEAN			26.59	3.62	7.34	1.42	0.55	0.86	
LOG MEAN			1.42	0.56	0.87				
STD DEV			5.76	0.48	1.20	0.09	0.06	0.07	
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN \pm OR- 2*STD.DEV.)									

TABLE E-3

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION KON

STATION 6 KON 17 - 44 SEC, EDIT NO. 12, SEISMOMETER OUTPUT									
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO	AMP PER
1	72	306	39.72	6.02	6.60	1.60	0.78	0.82	28
2	72	307	63.22	10.18	6.21	1.80	1.01	0.79	20
3	72	308	40.33	6.96	5.79	1.61	0.84	0.76	20
4	72	309	33.62	5.70	5.90	1.53	0.76	0.77	24
5	72	310	50.58	6.95	7.28	1.70	0.84	0.86	24
6	72	316	65.11	9.44	6.90	1.81	0.97	0.84	24
7	72	318	48.50	7.29	6.65	1.69	0.86	0.82	20
8	72	321	50.18	7.00	7.17	1.70	0.85	0.86	24
9	72	322	61.54	9.64	6.38	1.79	0.98	0.81	24
10	72	323	40.19	5.53	7.27	1.60	0.74	0.86	24
11	72	325	38.06	6.56	5.80	1.58	0.82	0.76	20
12	72	326	30.42	5.41	5.62	1.48	0.73	0.75	20
13	72	327	30.20	4.34	6.96	1.48	0.64	0.84	24
14	72	331	50.26	7.72	6.51	1.70	0.89	0.81	20
15	72	333	64.85	10.52	6.16	1.81	1.02	0.79	20
16	72	335	28.68	5.13	5.59	1.46	0.71	0.75	20
17	72	359	39.42	6.09	6.47	1.60	0.78	0.81	24
18	72	364	46.42	6.88	6.75	1.67	0.84	0.83	24
19	73	2	53.38	9.34	5.72	1.73	0.97	0.76	28
20	73	3	51.46	7.08	7.27	1.71	0.85	0.86	28
21	73	6	25.96	4.07	6.38	1.41	0.61	0.80	28
22	73	8	31.67	4.17	7.59	1.50	0.62	0.88	24
23	73	11	48.53	7.30	6.65	1.69	0.86	0.82	28
24	73	13	39.07	5.89	6.63	1.59	0.77	0.82	20
25	73	21	35.27	6.15	5.73	1.55	0.79	0.76	24
26	73	22	31.93	4.97	6.42	1.50	0.70	0.81	24
27	73	23	35.49	5.73	6.19	1.55	0.76	0.79	24
28	73	26	38.40	6.01	6.39	1.58	0.78	0.81	20
29	73	27	36.19	5.15	7.03	1.56	0.71	0.85	24
30	73	28	46.12	6.09	7.57	1.66	0.78	0.88	24
31	73	29	53.36	7.39	7.22	1.73	0.87	0.86	20
32	73	30	59.13	8.62	6.86	1.77	0.94	0.84	24
MEAN			43.98	6.73	6.55	1.63	0.81	0.81	
LOG MEAN			1.64	0.83	0.82				
STD DEV			11.21	1.71	0.58	0.11	0.11	0.04	
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN +OR- 2*STD.DEV.)									

TABLE E-4
INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION KIP
(PAGE 1 OF 2)

STATION 8 KIP 17 - 44 SEC, EDIT NO. 8, SEISMOMETER OUTPUT									
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO	AMP PER
1	72	306	53.25	7.91	6.73	1.73	0.90	0.83	20
2	72	307	44.54	6.93	6.43	1.65	0.84	0.81	20
3	72	308	44.23	7.19	6.15	1.65	0.86	0.79	20
4	72	309	55.34	6.91	8.01	1.74	0.84	0.90	28
5	72	310	48.82	6.08	8.03	1.69	0.78	0.90	24
6	72	311	35.73	6.29	5.68	1.55	0.80	0.75	24
7	72	312	62.74	9.55	6.57	1.80	0.98	0.82	20
8	72	316	39.98	6.60	6.06	1.60	0.82	0.78	24
9	72	318	48.28	8.67	5.57	1.68	0.94	0.75	24
10	72	319	54.94	9.37	5.86	1.74	0.97	0.77	20
11	72	322	59.22	8.57	6.91	1.77	0.93	0.84	24
12	72	323	49.96	7.12	7.02	1.70	0.85	0.85	20
13	72	326	35.62	6.37	5.59	1.55	0.80	0.75	24
14	72	327	32.75	5.30	6.18	1.52	0.72	0.79	24
15	72	328	45.38	6.84	6.63	1.66	0.84	0.82	20
16	72	333	63.49	8.90	7.13	1.80	0.95	0.85	24
17	72	334	51.86	7.33	7.08	1.71	0.87	0.85	28
18	72	335	49.92	7.41	6.74	1.70	0.87	0.83	24
19	72	340	59.54	9.31	6.40	1.77	0.97	0.81	24
20	72	343	31.67	5.80	5.46	1.50	0.76	0.74	32
21	72	344	39.49	5.16	7.65	1.60	0.71	0.88	28
22	72	346	38.95	6.49	6.00	1.59	0.81	0.78	32
23	72	347	49.49	7.52	6.58	1.69	0.88	0.82	20
24	72	349	65.19	9.20	7.09	1.81	0.96	0.85	24
25	72	350	36.63	4.81	7.62	1.56	0.68	0.88	24
26	72	352	41.30	7.24	5.70	1.62	0.86	0.76	24
27	72	353	46.40	5.66	8.20	1.67	0.75	0.91	24
28	72	354	39.90	6.70	5.96	1.60	0.83	0.77	24
29	72	355	33.97	5.66	6.00	1.53	0.75	0.78	24
30	72	356	44.49	6.29	7.07	1.65	0.80	0.85	24
31	72	361	47.52	7.56	6.29	1.68	0.88	0.80	24
32	72	362	46.22	6.25	7.40	1.66	0.80	0.87	24
33	73	6	41.93	5.21	8.05	1.62	0.72	0.91	28

TABLE E-4

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION KIP
(PAGE 2 OF 2)

STATION			8 KIP 17 - 44 SEC, EDIT NO.			8, SEISMOMETER OUTPUT			
MO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO	AMP PER
34	73	7	45.09	5.54	8.14	1.65	0.74	0.91	28
35	73	10	47.73	6.21	7.69	1.68	0.79	0.89	28
36	73	13	35.61	5.55	6.42	1.55	0.74	0.81	24
37	73	14	30.41	4.74	6.42	1.48	0.68	0.81	20
38	73	15	41.95	5.13	8.18	1.62	0.71	0.91	20
39	73	17	41.00	5.41	7.58	1.61	0.73	0.88	20
40	73	19	33.83	5.28	6.41	1.53	0.72	0.81	28
41	73	20	34.69	5.82	5.96	1.54	0.76	0.78	20
42	73	21	54.04	7.50	7.21	1.73	0.88	0.86	20
43	73	24	41.10	5.42	7.58	1.61	0.73	0.88	20
44	73	25	32.75	5.20	6.30	1.52	0.72	0.80	24
45	73	27	37.11	6.23	5.96	1.57	0.79	0.78	24
46	73	28	31.77	5.23	6.07	1.50	0.72	0.78	24
47	73	29	50.11	6.86	7.30	1.70	0.84	0.86	20
48	73	31	51.56	8.38	6.15	1.71	0.92	0.79	28
MEAN			44.74	6.69	6.73	1.64	0.82	0.83	
LOG MEAN			1.65	0.82	0.83				
STD DEV			9.05	1.33	0.80	0.09	0.08	0.05	
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN +CR- 2*STD.DEV.)									

TABLE E-5

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION ALQ

STATION 9 ALQ 17 - 44 SEC, EDIT NO. 12, SEISMOMETER OUTPUT									
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO	AMP PER
1	72	306	35.64	5.73	6.22	1.55	0.76	0.79	24
2	72	307	36.11	5.66	6.38	1.56	0.75	0.80	24
3	72	308	46.23	7.03	6.58	1.66	0.85	0.82	24
4	72	309	42.95	6.66	6.45	1.63	0.82	0.81	20
5	72	311	57.85	9.85	5.87	1.76	0.99	0.77	20
6	72	314	51.75	7.42	6.97	1.71	0.87	0.84	20
7	72	316	33.41	5.69	5.87	1.52	0.76	0.77	20
8	72	318	49.21	7.81	6.30	1.69	0.89	0.80	20
9	72	319	38.32	6.70	5.72	1.58	0.83	0.76	24
10	72	323	42.83	7.36	5.82	1.63	0.87	0.76	24
11	72	326	53.11	9.45	5.62	1.73	0.98	0.75	20
12	72	337	48.56	7.81	6.22	1.69	0.89	0.79	32
13	72	342	49.74	7.29	6.82	1.70	0.86	0.83	24
14	72	343	43.34	6.22	6.97	1.64	0.79	0.84	20
15	72	344	32.39	5.40	6.00	1.51	0.73	0.78	24
16	72	345	33.20	5.55	5.98	1.52	0.74	0.76	20
17	72	346	55.36	8.08	6.85	1.74	0.91	0.84	24
18	72	350	36.30	6.19	5.86	1.56	0.79	0.77	24
19	72	353	43.29	6.46	6.70	1.64	0.81	0.83	20
20	72	354	39.57	6.13	6.46	1.60	0.79	0.81	28
21	72	356	42.34	7.10	5.96	1.63	0.85	0.78	20
22	72	358	48.25	7.50	6.43	1.69	0.88	0.81	20
23	72	362	58.21	9.57	6.08	1.76	0.98	0.78	20
24	73	2	39.23	6.14	6.39	1.59	0.79	0.81	24
25	73	7	29.64	4.77	6.21	1.47	0.68	0.70	24
26	73	13	41.91	6.90	6.07	1.62	0.84	0.78	24
27	73	14	36.38	6.03	6.03	1.56	0.78	0.78	24
28	73	16	61.92	9.92	6.24	1.79	1.00	0.80	28
29	73	17	59.18	9.56	6.19	1.77	0.98	0.79	20
30	73	21	46.38	7.27	6.38	1.67	0.86	0.80	28
MEAN			44.42	7.11	6.26	1.64	0.84	0.80	
LOG MEAN			1.65	0.85	0.80				
STD DEV			8.76	1.41	0.36	0.09	0.08	0.03	
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN +CR- 2*STD.DEV.)									

TABLE E-6

INTERMEDIATE VALUES USED IN DETECTION CAPABILITY ESTIMATION
FROM NOISE: STATION ZLP

STATION 10 ZLP 17 - 44 SEC, EDIT NO. 10, SEISMOMETER OUTPUT								
NO	YR	DAY	AMP	RMS	AMP/ RMS	LOG AMP	LOG RMS	LOG RATIO
1	72	325	24.73	3.78	6.54	1.39	0.58	0.82
2	72	326	25.71	4.19	6.14	1.41	0.62	0.79
3	72	328	35.01	5.44	6.44	1.54	0.74	0.81
4	72	329	42.65	5.58	7.64	1.63	0.75	0.88
5	72	334	26.50	4.65	5.70	1.42	0.67	0.76
6	72	335	21.48	3.60	5.97	1.33	0.56	0.78
7	72	343	51.01	6.59	7.74	1.71	0.82	0.89
8	72	344	44.39	5.71	7.77	1.65	0.76	0.89
9	72	345	34.05	5.20	6.55	1.53	0.72	0.82
10	72	350	28.81	4.17	6.91	1.46	0.62	0.84
11	72	351	44.19	6.50	6.80	1.65	0.81	0.83
12	72	354	42.10	6.32	6.65	1.62	0.80	0.82
13	72	355	41.72	5.85	7.13	1.62	0.77	0.85
14	72	356	28.62	4.82	5.94	1.46	0.68	0.77
15	72	358	19.13	3.22	5.94	1.28	0.51	0.77
16	72	359	20.87	3.57	5.85	1.32	0.55	0.77
17	73	13	28.20	4.02	7.01	1.45	0.60	0.85
18	73	14	23.52	4.13	5.69	1.37	0.62	0.76
19	73	19	30.57	4.98	6.14	1.49	0.70	0.79
20	73	22	26.18	4.05	6.46	1.42	0.61	0.81
21	73	24	34.97	4.62	7.57	1.54	0.66	0.88
MEAN			32.11	4.81	6.60	1.49	0.67	0.82
LOG MEAN			1.51	0.68	0.82			
STD DEV			9.12	1.01	0.68	0.12	0.09	0.04
ALL RATIO AND LOG VALUES ARE WITHIN (MEAN +OR- 2*STD.DEV.)								